

Chapter 23 - Potential

Voltage (potential) is the analogue of **water pressure** while **current** is the analogue of **flow of water** in say gal/min or Kg/s ...

Think of a potential as the words sounds like – **the potential to do work**
Potential Energy

We define potential as a the work per unit charge in moving a charge from point a to b

Along some path that we choose

In a static field (not time varying) this energy is conservative
No matter what path you take you always get the same energy

In static gravity this is the same – **it is a conservative field**

Potential and Line Integrals in Space

- Mathematically this is a line integral along curve C from a to b.
- Note that in a closed line integral from point a back to point a the total energy is ZERO in a conservative field. Again a static E field is a conservative field

$$\varphi_{\mathbf{E}} = - \int_C \mathbf{E} \cdot d\boldsymbol{\ell} ,$$

Work and Electric Fields

- Work (energy) is then potential (voltage) times charge moved from a to b

$$U_{\mathbf{E}} = q \varphi .$$

For such a field we can show mathematically that we can express the E field as the gradient of the potential. We use a minus as a convention for **work done on the charge** by the field vs **work the charge does on the field**

$$\mathbf{E} = -\nabla \varphi_{\mathbf{E}} .$$

Poisson Equation and Laplacian

- If we take the Divergence of the E field we get the Poisson Equation

$$\nabla \cdot \mathbf{E} = \nabla \cdot (-\nabla \varphi_{\mathbf{E}}) = -\nabla^2 \varphi_{\mathbf{E}} = \rho / \epsilon_0 ,$$

The “Del squared” is called the Laplacian operator = ∇^2
Note the Laplacian operating on a scalar function is a scalar

$$\Delta f = \nabla^2 f = \nabla \cdot \nabla f, \quad \Delta f = \sum_{i=1}^n \frac{\partial^2 f}{\partial x_i^2} .$$

Potential of a Point Charge

$$\varphi_{\mathbf{E}} = \frac{1}{4\pi\epsilon_0} \frac{q}{r},$$

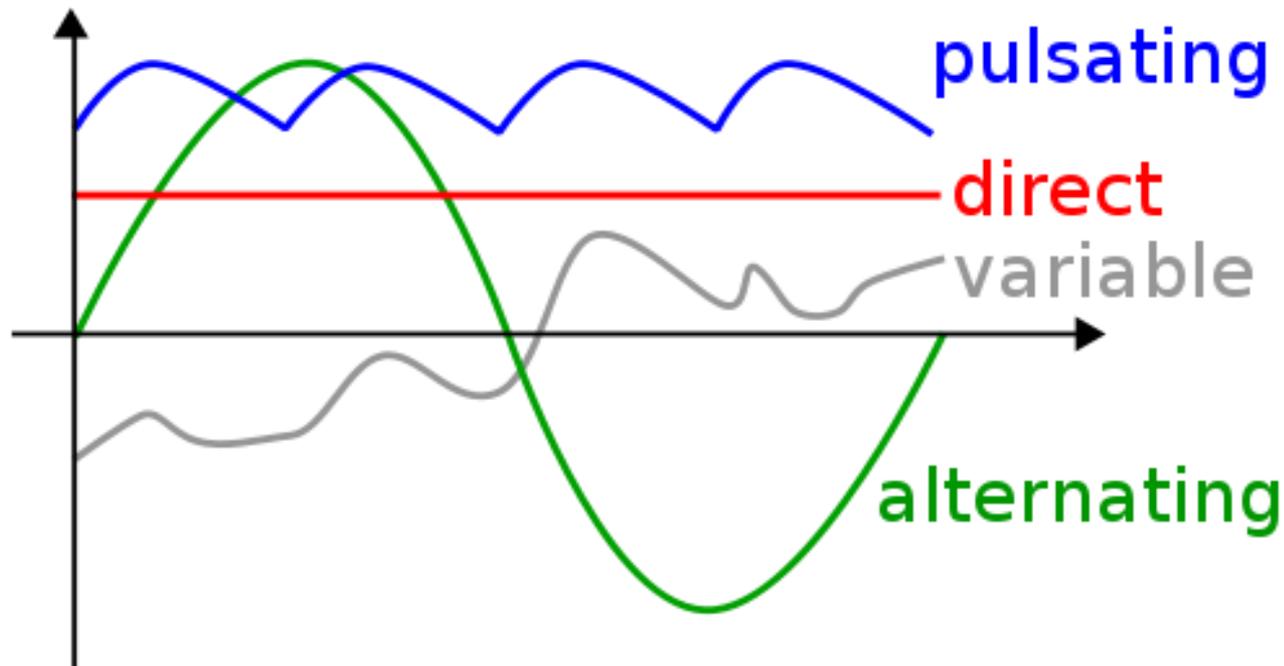
$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{\mathbf{r}} \quad (1)$$

Note that the potential goes as $1/r$ while the E field goes as $1/r^2$

Types of Voltages AC, DC, Pulse, Variable

AC = Alternating Current

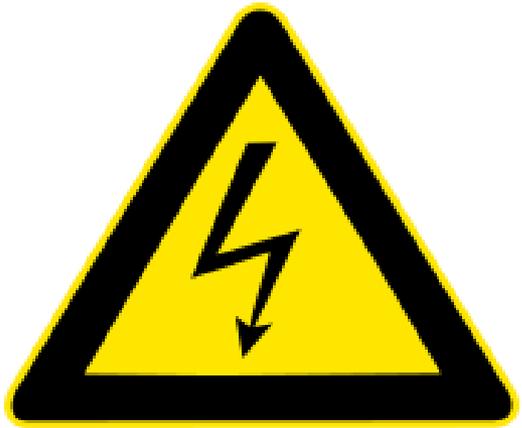
DC = Direct Current



Digital Volt Meter and International HV Symbol



DVM

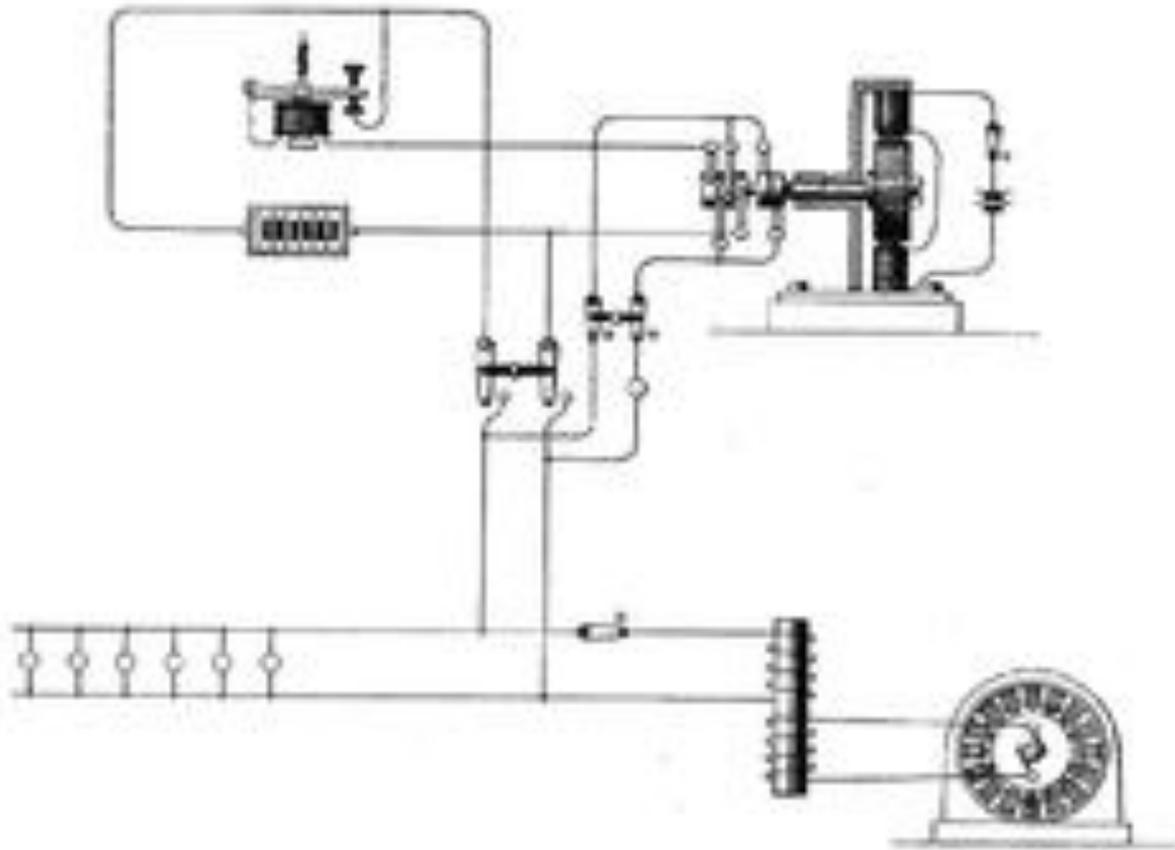


International Symbol of High Voltage

AC vs DC Power

- This is a curious topic with a rich history – largely forgotten today
- Our transmitted power is almost all AC today largely because it is easy to convert low to high to low voltage in transformers.
- However it is feasible and even desirable in many case to use DC
- Historically there was not simple and efficient way to transform DC voltages
- DC was advocated by Thomas Edison
- AC by Nikola Tesla and George Westinghouse (yes)

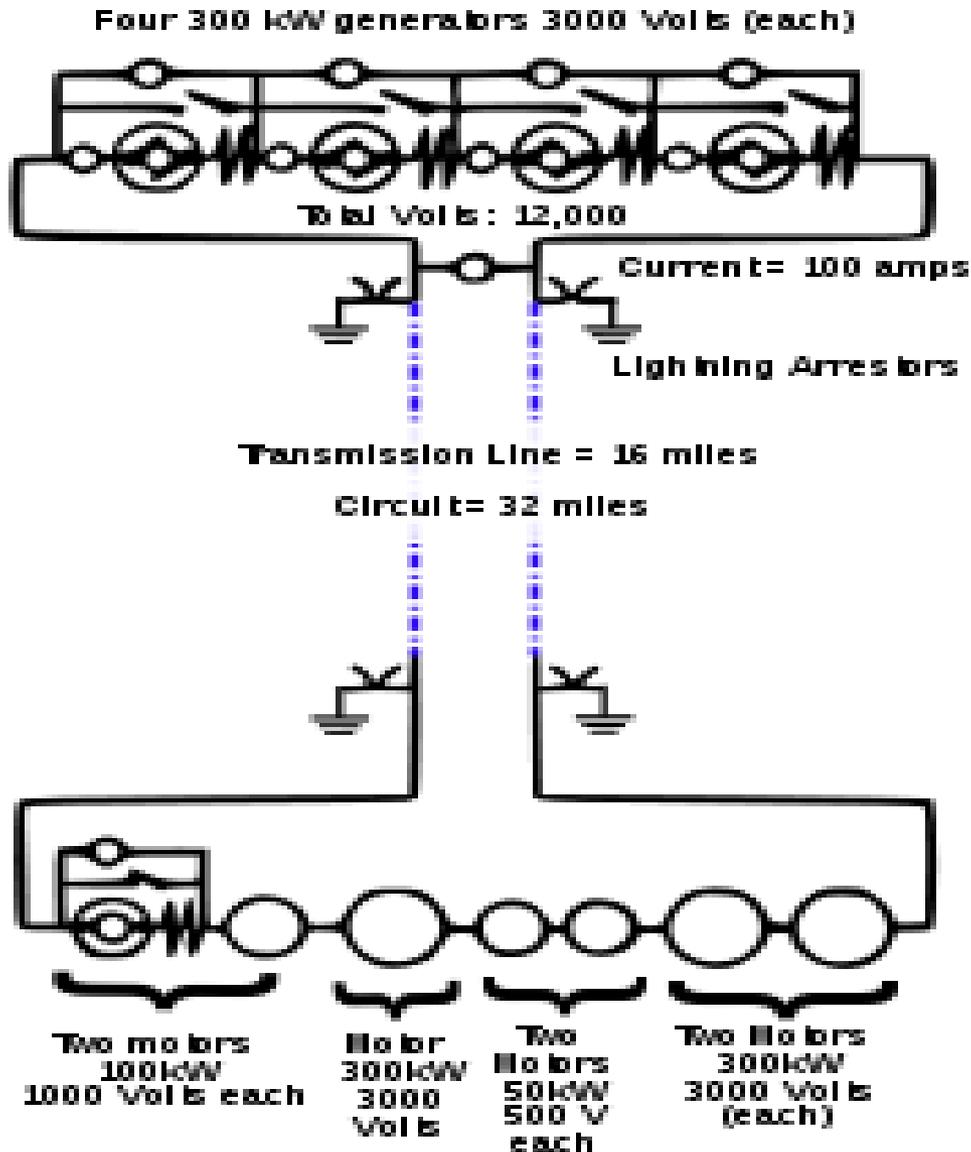
AC Generators go back ~ 150 years Westinghouse 1887



AC and DC

- AC largely won out early in the 20th Century
- But DC was not abandoned
- 1882 [Miesbach-Munich Power Transmission](#), 2.5 Kw
- 1889 [Italy](#) by the *Acquedotto De Ferrari-Galliera* up to 630 Kw at 14 Kv DC over 120 Km distance
- [Moutiers-Lyon](#) system transmitted ~ 10 MW at 150 KV of hydroelectric power a distance of 124 miles – used from 1906 to 1936
- Most of these early DC systems used series DC generators – Rene Thury

Thury System for HVDC



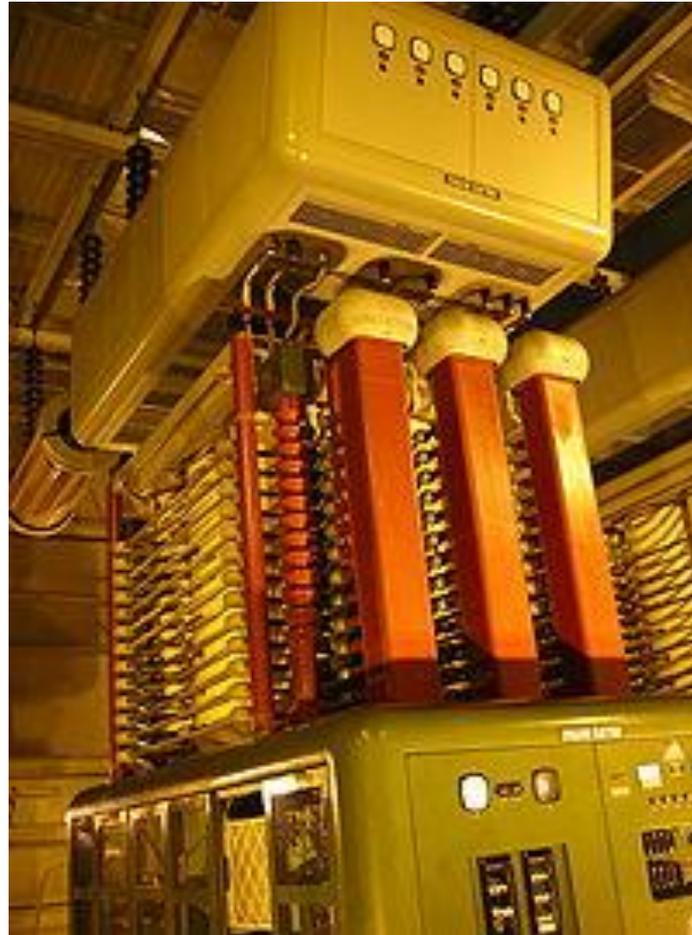
Modern HVDC Systems

- Modern systems became possible starting in the 1930 – 40' and much more so today with modern semiconductor switches
- [mercury arc valves](#) – Early “tube” type switches
- Semiconductor devices- [thyristors](#), [insulated-gate bipolar transistors](#) (IGBTs), [MOSFETs](#) (power metal–oxide–semiconductor field-effect transistors)
- [gate turn-off thyristors](#) (GTOs)

Some more History

- 1941 [Elbe-Project](#) 60 MW 200 KV 115 Km underground DC line for Berlin – buried so less of a target for Allied bombing during WW-II
- Never finished due to end of war
- Dismantled and taken to USSR and used
- It used Mercury Arc Valves (MAV) to convert DC
- 1954 Sweden to Gotland (Island)
- 1975 to 2000, line-commutated converters (LCC) using [thyristor](#)
- Hundreds of deep sea HVDC systems built
- Some HVDC is now 1 mega Volt
- Future of DC is for long distance high power applications for now
- And of course low voltage systems using batteries – ie cell phones
- Solar and Wind power are more naturally DC but current grid ties convert them to AC

1971 Mercury Arc valve

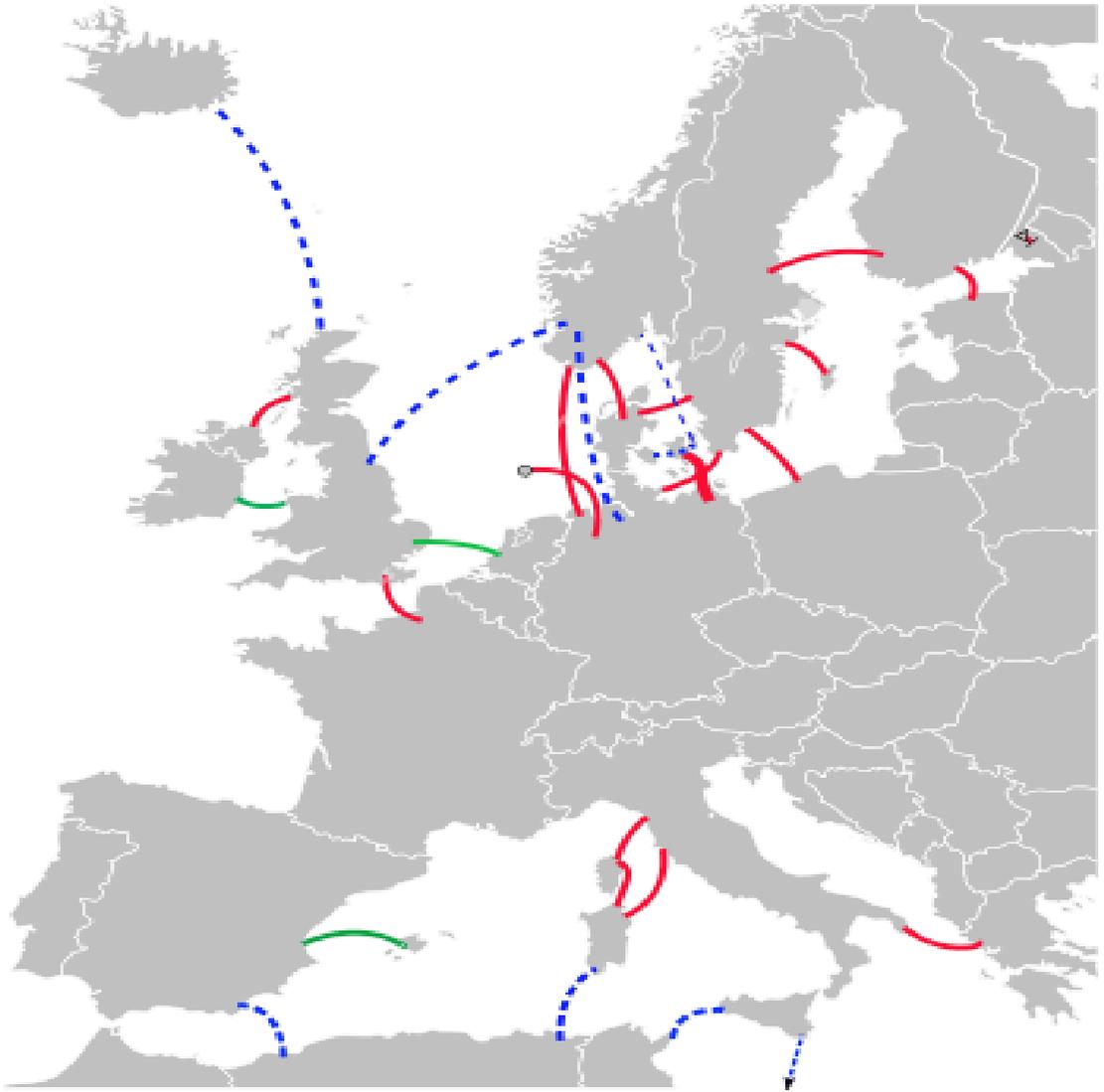


Some Existing and Planned HVDC Systems in Europe

Red – Existing

Green – Under Construction

Blue - Planned



Why would we use HVDC

- AC radiates energy due to accelerating changes
- It is a low frequency antenna
- Ever driven under a HVAC lines with an AM radio on?
- Ever lit up a fluoescent bulb under a HVAC line?
- 3% per 1000 Km
- Capacitance is an issue for very long runs under water where the normally one cable is buried with at least two lines – close to each other – high capacitance
- [Undersea cables](#) - high capacitance cause additional AC losses.
- 250 km [Baltic Cable](#) between [Sweden](#) and [Germany](#)^[12]
- 600 km [NorNed](#) cable between Norway and the Netherlands
- Skin effect – at 60 Hz the current only penetrates about 1 cm into wire – need to strand cable to be effective in AC
- Skin depth δ goes inversely as square root of freq – high freq do not penetrate very deep into metal
- This is very useful in mirror coating where very thin layers of metal reflect radiation
- “Space blanket” is one example – few hundred Angstroms of Aluminum on mylar

Generalization of Potential in Electrodynamics vs Electrostatics

- In Chapter 23 we are assuming static electric charges – ie they do not move
- We will see that the E field and the B field
- Like Gravity move at the speed of light
- This means if you move a set of charges
- You will not feel the effects of the moved charges instantaneous. Just as you will not SEE them as having moved.

What does this mean in practice

- It can be shown using relativistic electrodynamics that **changing E** fields generate B (magnetic) fields and that in fact there are no true B fields known.
- As far as we can tell all magnetic fields are relativistically transformed E fields
- It only appears as if there are magnetic fields
- It depends on our frame of reference
- It is **relative**
- **We will discuss this more later**

Generalization of the potential

- In the static charge case the potential (work done) is conservative – ie the same between two given points **NO MATTER WHAT PATH IS TAKEN TO GO BETWEEN THEM**
- But in dynamics (moving charges this is no longer the case
- If we use Stokes Theorem that relates a close **LINE** integral over a curve in space to the **CURL** of a surface integral **BOUNDED** by that curve.
- The line integral is closed the surface intergal is open

Dynamics and Conservative Fields

- In a conservative field (static charges for example) the closed line integral of the field is ALWAYS ZERO
- In a non-conservative field this is not necessarily true
- One simple example would be the work on a ball thrown in the gravitation field
- If you start at the ground and throw it up
- When it comes down it has the same energy you threw it up at even if it does not land at the same spot as long as it is at the same LEVEL
- Imagine you suddenly moved the Earth after you threw the ball then this would no longer be true – this is dynamics
- Things move at finite speeds and so do the fields

- We can introduce a new vector field that modifies the electric field which is conservative
- Even in dynamics
- The field is:

$$\mathbf{F} = \mathbf{E} + \frac{\partial \mathbf{A}}{\partial t}$$

- One can show from Stokes Theorem that conservative fields can always be written as Gradients of a scalar field
- Before this other scalar field was the voltage (potential)

New Fields

- We can then write a generalized E field in the full dynamics case as:

$$\mathbf{E} = -\nabla\varphi - \frac{\partial \mathbf{A}}{\partial t},$$

But keep in mind the E field is not conservative

$$\int_a^b \mathbf{E} \cdot d\boldsymbol{\ell} \neq \varphi(b) - \varphi(a), ,$$

More Generally

- The “B” field is the Curl of the A field
- A is called the Vector potential

$$\mathbf{B} = \nabla \times \mathbf{A} ,$$

- Unless there are magnetic charges (monopoles) then the Divergence of the B field is always ZERO. This is also a consequence of relativity.
- This is Maxwell Eq #2

Gauge Choice

- The Curl of a Gradient is always ZERO
- Hence you can add a Gradient term to the vector potential \mathbf{A} and not change \mathbf{B} since
$$\mathbf{B} = \nabla \times \mathbf{A} ,$$
- This brings up the fact that we are free to choose
- A “gauge field” in which we have a number of common choice. Two are:
- Lorentz Gauge $\nabla \cdot \mathbf{A} = -\frac{1}{c^2} \frac{\partial \varphi}{\partial t} .$
- Coulomb Gauge $-\nabla^2 \varphi = \rho / \epsilon_0 ,$
- This is similar to the fact the voltage is only defined up to a constant