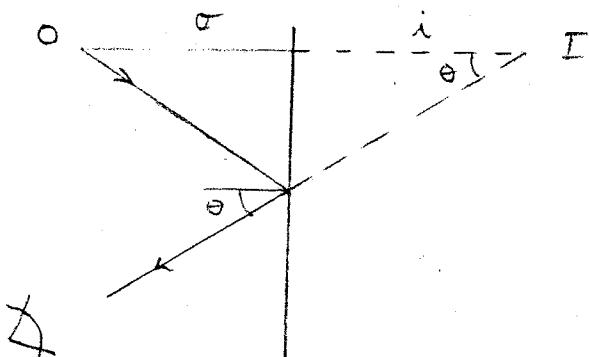


Minors

I



Minor

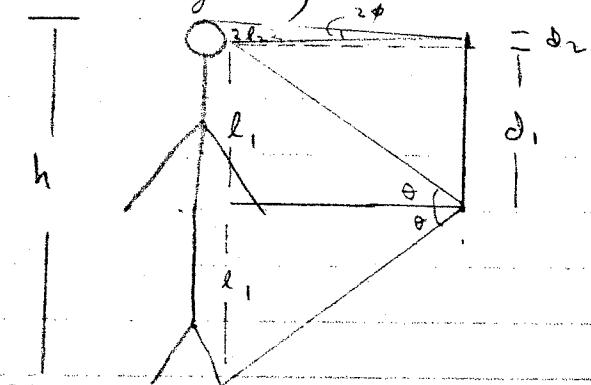
I is virtual image

$$\sigma = -i \quad i - \text{neg since image} \\ \rightarrow \text{virtual}$$

Minor interchanges right & left
(reverses parity), reverses spin orientation

II

Size of minor to view entire body



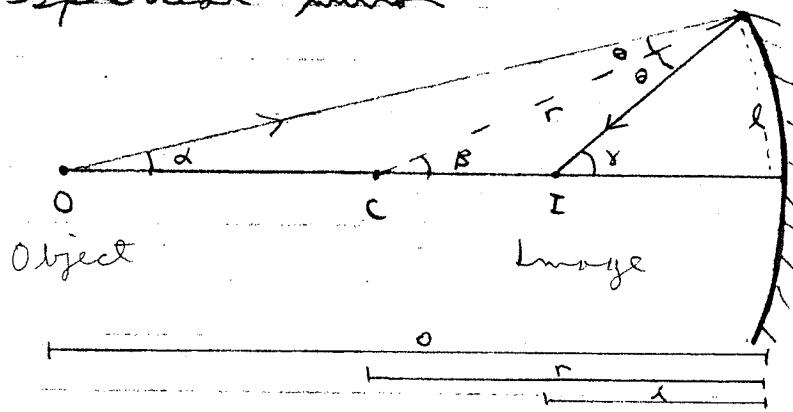
$$l_1 = d_1$$

$$l_2 = d_2$$

$$2l_2 + 2l_1 = h = 2(d_1 + d_2) = 2 \times \text{minor eye}$$

Spherical mirror

from book



C = center of sphere

r = radius of sphere

$$\beta = \alpha + \theta \quad 2\beta = 2\alpha + 2\theta$$

$$\gamma = \alpha + 2\theta \quad \gamma = \alpha + 2\theta$$

$$\alpha + \gamma = 2\beta \leftarrow 2\beta - \gamma = \alpha$$

$$\alpha \approx \frac{l}{o}$$

$$\beta \approx \frac{l}{r}$$

$$\gamma \approx \frac{l}{i}$$

for small angles or

$$\alpha + \gamma = 2\beta \rightarrow \frac{1}{o} + \frac{1}{i} = \frac{2}{r}$$

o, i, r positive if on real (illuminating) side of mirror

o, i, r negative if on virtual side

When $o \rightarrow \infty \quad \frac{1}{i} = \frac{2}{r} \quad i = \frac{r}{2}$

i is called the focal length f

$f = \frac{r}{2} - I$ is focal point the

Focusing at $\frac{r}{2}$, radius!

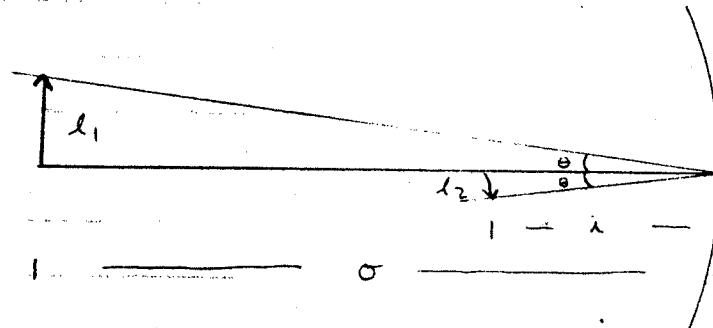
$$\frac{1}{o} + \frac{1}{i} = \frac{2}{f}$$

Note if $o = r$ then $i = r$

f is positive for concave mirrors
(image is real)

f is negative for convex mirrors
(virtual image)

Magnification



$$\frac{1}{d} + \frac{1}{i} = \frac{1}{f}$$

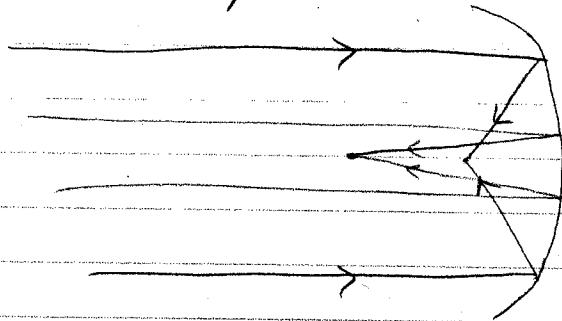
$$\tan \theta = \frac{l_1}{d} = \frac{l_2}{i}$$

$$m = -\frac{l_2}{l_1} = -\frac{i}{d}$$

(negative because of image inversion) magnification

Note concave mirror inverts image

Note: A spherical mirror does not focus parallel rays to a single point - Focus is smeared.
Called Spherical aberration

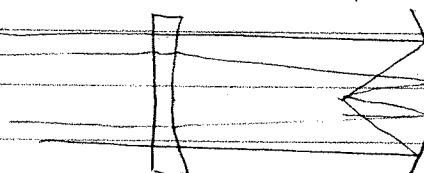


What surface does focus to a point?

Answer a Parabola

Many telescopes use parabolae

Could also use spherical mirror + connector plate

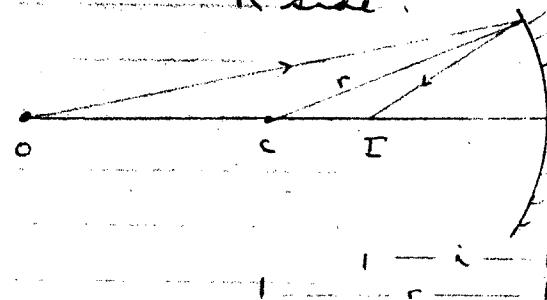


Spherical mirrors are easier to make

Examples

"R" side.

1)



"V" side

Real image I

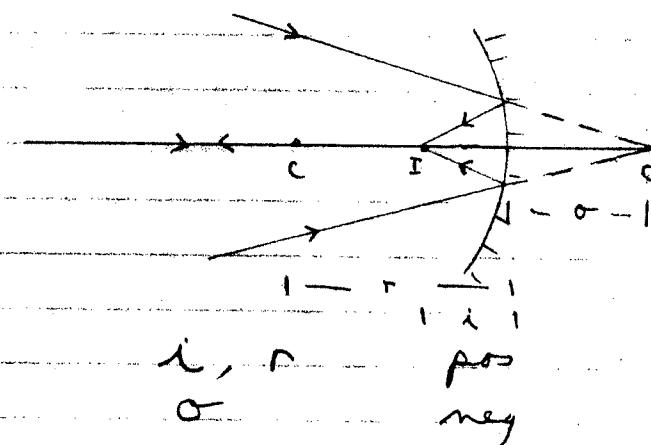
Real object O

Center of curvature
R side

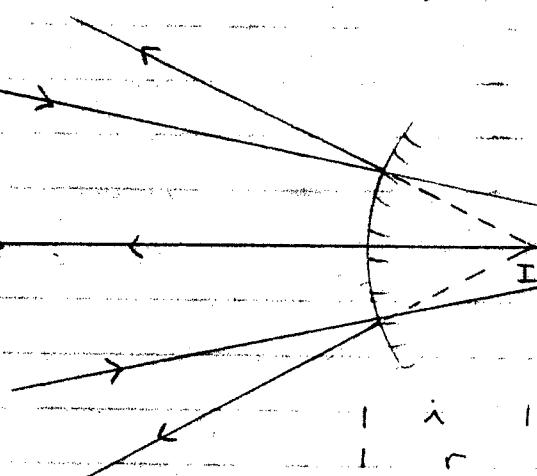
$$\frac{1}{i} + \frac{1}{o} = \frac{1}{f}$$

$i, o > f$ pos

2)



Real image I
Virtual object O



Virtual image I
Virtual Obj
Center of curv
on V side

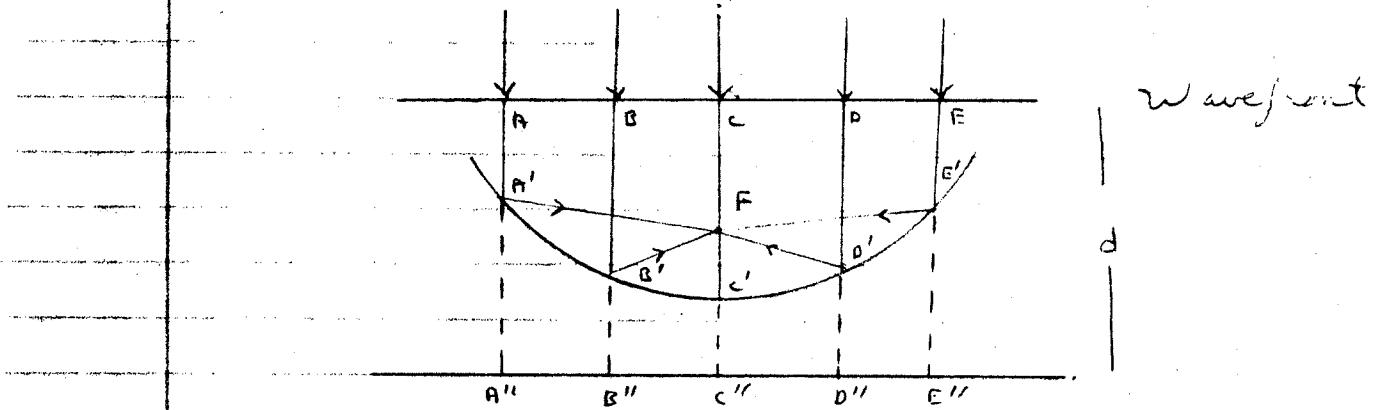
$$\frac{1}{i} + \frac{1}{o} = \frac{1}{f}$$

$$f = \frac{2}{n}$$

$i, o < f$ neg.

3) Telescope - What is surface function.

Imaging \rightarrow paths all have same (minimum) time



Same minimum time ('first order large $\Rightarrow 0$) $\rightarrow AA' + A'E = BB' + B'F = CC' + C'F = \dots$

$$\rightarrow d - (AA' + A'E) = d - (BB' + B'F) = \dots$$

$$\rightarrow B'A'' - A'E = B'B'' - B'F = \dots$$

adjust d such that $A'A'' - A'F = 0$

$$\text{then } B'A'' - A'F = B'B'' - B'F = \dots = 0$$

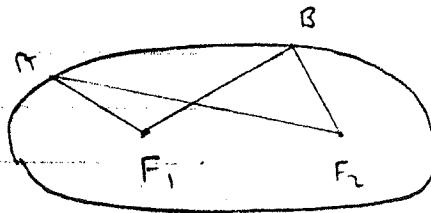
$$\rightarrow A'A'' = A'F \quad B'B'' = B'F \quad \dots$$

Such a curve is a parabola where F is the focus

"A parabola is the locus of points in a plane equidistant from a given point and a given line. The point is the focus and the line the directrix."

Surface of telescope = parabolic

4) Focusing in ellipse



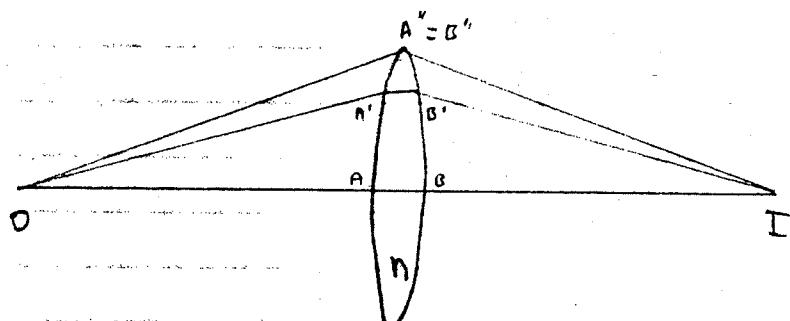
Focus if all times of transit are equal. i.e. if $F_1 A + A F_2 = F_1 B + B F_2$
such a curve is an ellipse.

Application

- 1) Laser - Pump flash lamp, etc
- 2) Sound concentrator
- 3) X-ray concentrator to burn weapons

5)

Lens



Focus if all times are equal

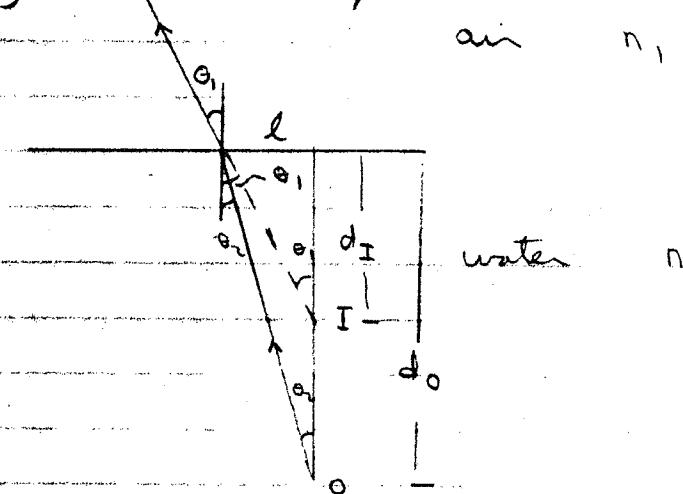
$$\begin{aligned} & \text{re. if } t_{OA} + t_{A'B'} + t_{B'I} \\ &= t_{O'A'} + t_{A'B'} + t_{B'I} \end{aligned}$$

$$= t_{OA''} + t_{A''B''}^{=0} + t_{B''I}$$

Is it even conceivable? Yes since
light goes slower in material by factor n

Examples

1) Looking into pool



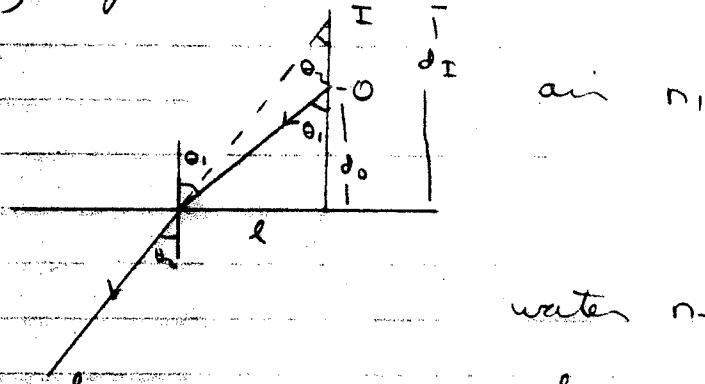
Object appears shallower than it is

$$d_I = \frac{l}{\tan \theta_1}, \quad d_0 = \frac{l}{\tan \theta_2}$$

$$\frac{d_0}{d_I} = \frac{\tan \theta_2}{\tan \theta_1} = \frac{\theta_2}{\theta_1} \sim \frac{n_1}{n_2} \sim 1.33 \text{ for small } \theta$$

Object appear 1.33 times closer

2) Looking from underwater into air



$$d_I = \frac{l}{\tan \theta_2}, \quad d_0 = \frac{l}{\tan \theta_1}$$

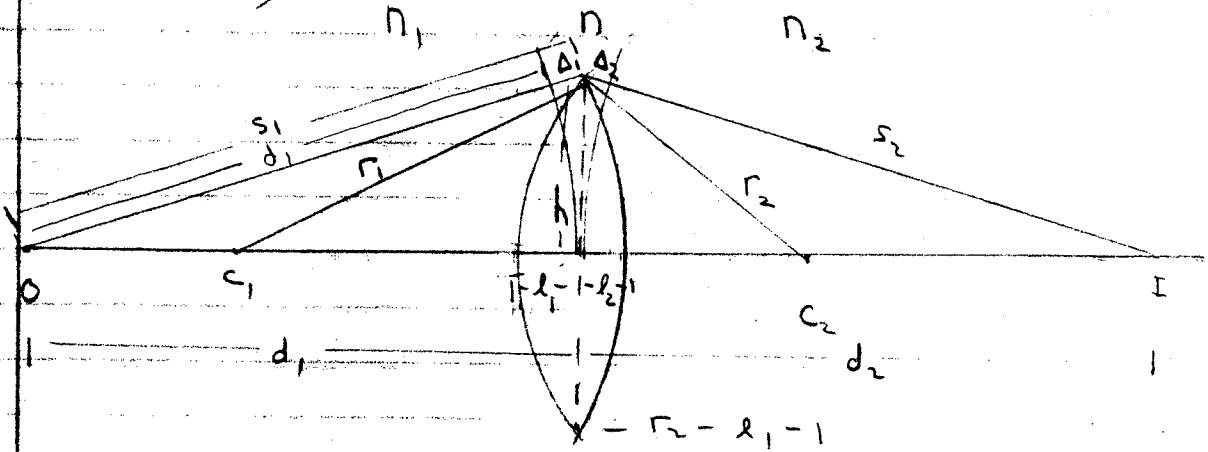
$$\frac{d_0}{d_I} = \frac{\tan \theta_2}{\tan \theta_1} \sim \frac{\theta_2}{\theta_1} \sim \frac{n_1}{n_2} \sim 1.33 \text{ for small } \theta,$$

Objects appear 1.33 times further

Lenses - spherical

Feynman II 27

1) By minimum time



$$\Delta_1 = s_1 - d_1 = \text{excess time } 1$$

$$\Delta_2 = s_2 - d_2 = " " " 2$$

$$s_1^2 = d_1^2 + h^2 \quad s_1^2 - d_1^2 = (s_1 - d_1)(s_1 + d_1) = h^2$$

$$h^2 = \Delta_1 (s_1 + d_1) \sim 2s_1 \Delta_1 \quad d_1 \sim s_1$$

similarly $h^2 \sim 2s_2 \Delta_2 \quad d_2 \sim s_2$

$$\Delta_1 \sim \frac{h^2}{2s_1}, \quad \Delta_2 \sim \frac{h^2}{2s_2}$$

Foci of transit times are equal

$$C t = n_1 s_1 + n_2 s_2$$

$$(r_2 - l_1)^2 + h^2 = r_2^2 = r_2^2 - 2l_1 r_2 + l_1^2 + h^2$$

$$l_1^2 \ll l_1 r_2 \quad h \sim \frac{h^2}{2r_2} \quad \text{also} \quad l_2 \sim \frac{h^2}{2r_1}$$

Calculate light travel time for direct path O I : Set equal to s_1, s_2

$$\text{Direct path } C t = n_1(d_1 - l_1) + n_2(d_2 - l_2) = n_1 s_1 + n_2 s_2 = n_1(d_1 + \Delta_1) + n_2(d_2 + \Delta_2)$$

$$\rightarrow (n - n_1) l_1 + (n - n_2) l_2 = n_1 \Delta_1 + n_2 \Delta_2$$

$$\rightarrow (n - n_1) \frac{h^2}{2r_2} + (n - n_2) \frac{h^2}{2r_1} = \frac{n_1 h}{2s_1} + \frac{n_2 h}{2s_2}$$

$$\rightarrow \frac{n-n_1}{r_2} + \frac{n-n_2}{r_1} = \frac{n_1}{s_1} + \frac{n_2}{s_2}$$

For a thin lens $s_1 \sim d_1 = o$
 $s_2 \sim d_2 = i$

$$\frac{n_1}{o} + \frac{n_2}{i} = \frac{n-n_1}{r_2} + \frac{n-n_2}{r_1} = \frac{n_1}{f_1} = \frac{n_2}{f_2}$$

$$\frac{n_1}{f_1} = \frac{n_2}{f_2} = \frac{n-n_1}{r_2} + \frac{n-n_2}{r_1} \quad \text{let } o, i \rightarrow \infty$$

f_1, f_2 - focal lengths

In air (vacuum) $n_1 = n_2 = 1$

$$\rightarrow s_1 = f_2 \quad \frac{1}{o} + \frac{1}{i} = \frac{1}{f} = (n-1)(\frac{1}{r_1} + \frac{1}{r_2})$$

In notation of Halliday & Resnick

$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f} = (n-1)(\frac{1}{r_1} - \frac{1}{r''})$$

lens makers formula

where r'' is neg for our case

$R_2, r'' = -r_1$ radius of second surface

$R_1, r' = r_2$ radius of first surface

In notation of Giancoli

$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f} = (n-1)(\frac{1}{R_1} + \frac{1}{R_2})$$

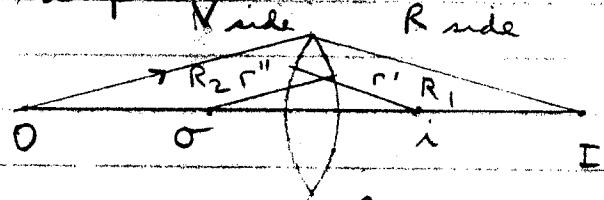
note $R_2 = -r_1, R_1 = r_2$
 in previous discussion

Sign conventions

positive δ, i	real object, real image
negative δ, i	virtual object, virtual image
positive r', r''	center of curvature on R side
negative r', r''	" " " " " V side

$r_2 \uparrow R, R_2 \uparrow r_1$

Examples



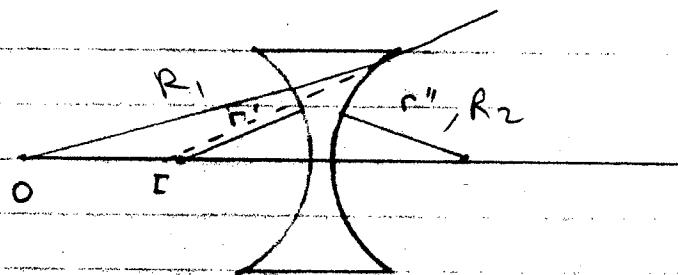
O real O +

I real I +

$R_2 r''$ on V $R_2 r''$ -

$R_1 r'$ on R $R_1 r'$ +

} F (+)



O real

O +

I virtual

I -

$R_2 r''$ on R

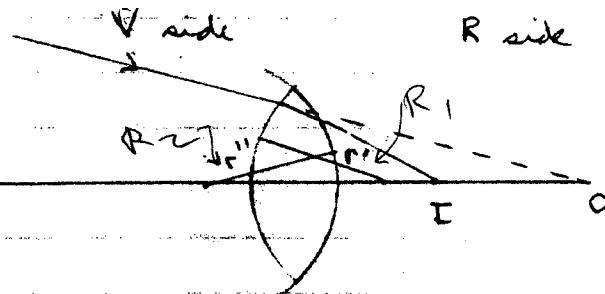
$R_2 r''$ +

$R_1 r'$ on V

$R_1 r'$ -

} F (-)

Halliday & Resnick
Giancoli



O virtual

I real

$R_2 r''$ on V

$R_1 r'$ on R

O -

i +

$R_2 r''$ -

$R_1 r'$ +

Focal length

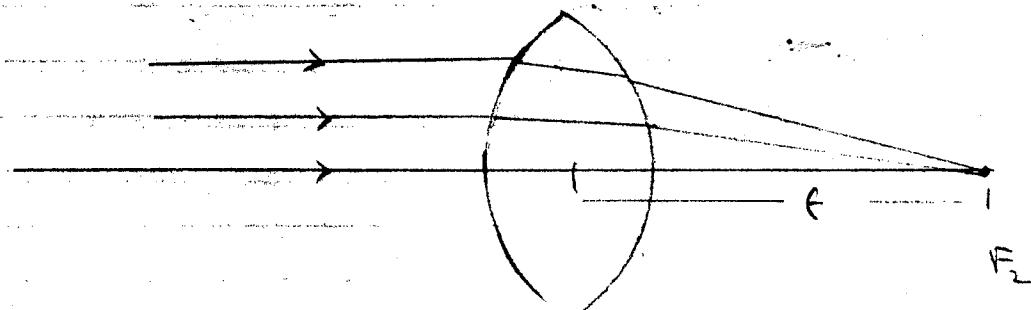
$$\frac{1}{f} = \frac{1}{n} - \frac{1}{R_1} - \frac{1}{R_2}$$

Thin lens

$$\frac{1}{f} = (n-1) \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

If $\theta = \infty$ ie parallel ray
then $i = f$ = focal length

For thin lenses the two
focal points F_1, F_2 are
equidistant from the lens if $n_1 = n_2$



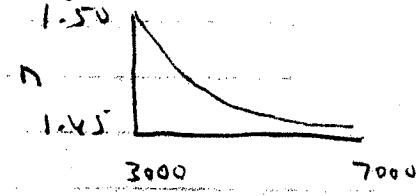
Aberations

1) Spherical

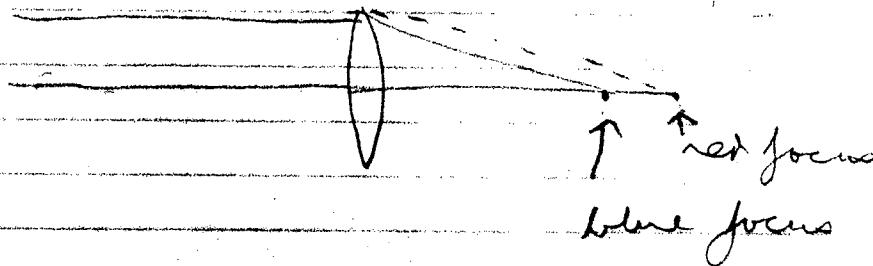
spherical surface is only an approximation. Actual surface is quartic (4th order eq).

2) Chromatic

Lens material is often (always) dispersive $n = n(\lambda)$ \rightarrow focal length depends on λ .



$$f = \left((n-1) \left(\frac{1}{r_1} - \frac{1}{r_2} \right) \right)^{-1}$$



Ray tracing

- 1) Ray parallel to axis goes through focus
- 2) Ray through center passes ~ unchanged
- 3) Ray through first focus emerges parallel to axis

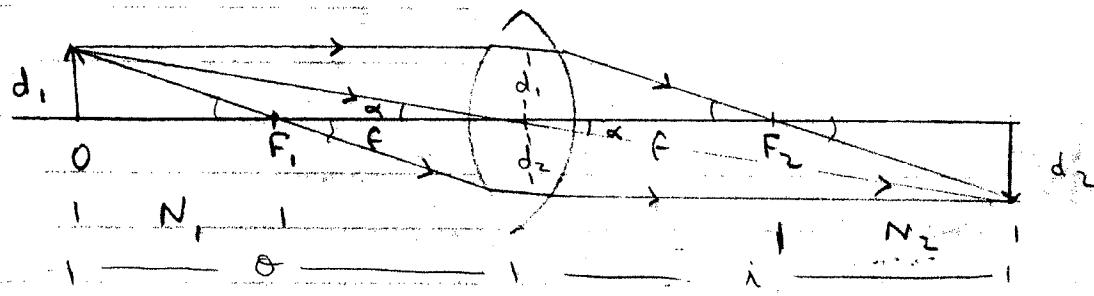


Image is inverted.

$$\frac{d_1}{o} = \frac{d_2}{f} \text{ lateral magnification } m = -\frac{d_2}{d_1} = -\frac{i}{o}$$

Newton's Equation for thin lenses from above

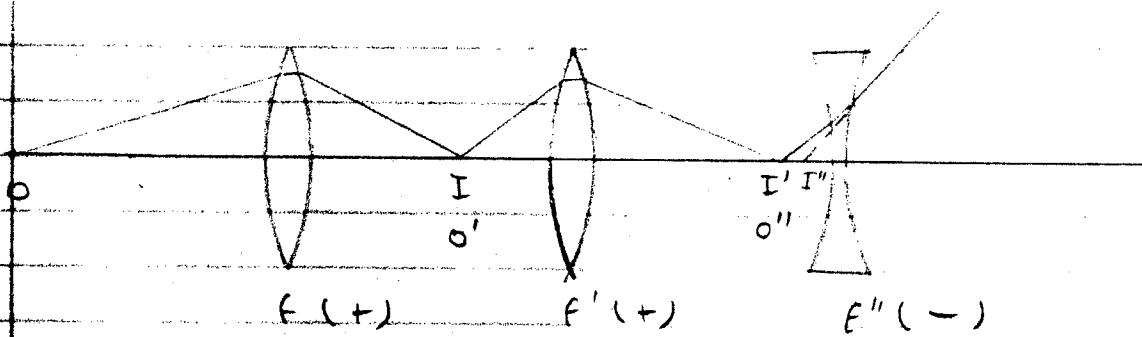
$$\frac{d_1}{N_1} = \frac{d_2}{f} \text{ and } \frac{d_2}{N_2} = \frac{d_1}{f}$$

$$f = N_1 \frac{d_2}{d_1} = N_2 \frac{d_1}{d_2}$$

$$\rightarrow f^2 = N_1 N_2$$

Compound Lenses

How do we treat compound lenses?
Solution : Find image due to first lens then use this as object for next image etc

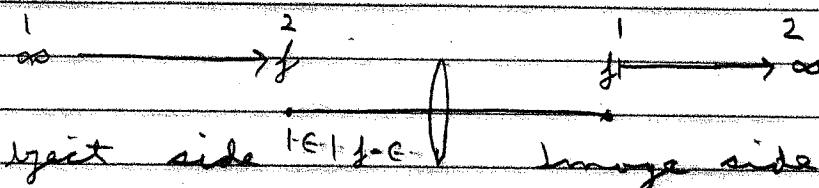


$$f_1 (+) \quad f_2 (+) \quad f_3 (-)$$

$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f_1} \quad \frac{1}{o'} + \frac{1}{i'} = \frac{1}{f_2} \quad \frac{1}{o''} + \frac{1}{i''} = \frac{1}{f_3}$$

$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f}$$

$$i = \left(\frac{1}{f} - \frac{1}{o} \right)^{-1}$$



$o = \infty$	$i = f$	$\alpha = f - e$
∞	f	$i = \frac{f(f-e)}{e}$
f	∞	
$f/2$	$-f$	
o	$-o$	
$f-e$	$-f(f-e)/e$	

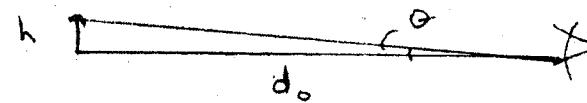
Power of lens
Dropter - definition

$$\text{Power} (\text{in dioptres}) = \frac{1}{f} \quad f \text{ - in meter}$$

Examples

i) Eyes + magnifier

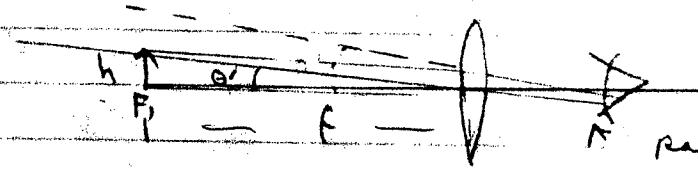
My eye can focus to 15 cm



$$d_o \sim 15 \text{ cm} \quad \theta = \frac{h}{d_o}$$

For my eye

Add one lens. Put object at focus of lens & rays leave lens parallel. so eye can relax what is magnification?



Recall: Ray thru a
is unchanged

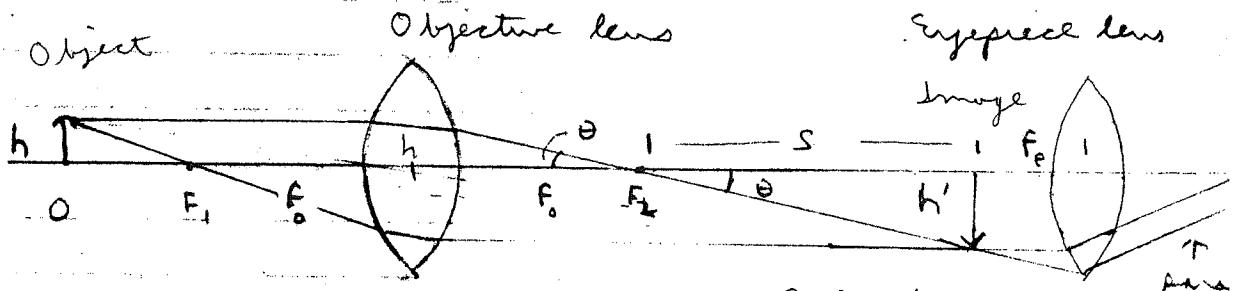
parallel rays

$$\theta' = \frac{h}{F} \quad m_o = \frac{\theta'}{\theta} = \frac{\frac{h}{F}}{\frac{h}{d_o}} = \frac{d_o}{F}$$

$m_o > 1$ if $f < d_o$

m_o - Angular magnification not lateral
as before - no image is formed by lens
only by eye

Microscope



f_o = objective lens focal length

F_1, F_2 focus of objective lens

$$M = \text{lateral image magnification} = -\frac{h'}{h}$$

f_e = eyepiece focal length

$$h' = s \tan \theta$$

$$h = f_o \tan \theta$$

$$m = -\frac{s}{f_o}$$

$$m_o = \frac{d_o}{f_o}$$

eyepiece focus
at image

eyepiece

paraxial

s - distance between
objective focus and
eyepiece focus

lateral mag of object

angular mag of image h'
from previous example

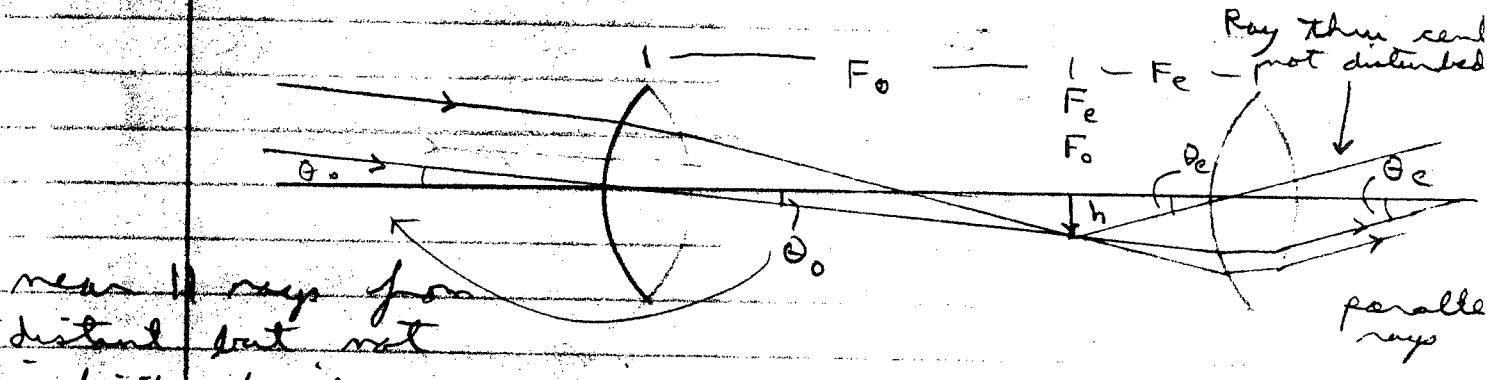
$$\text{Total magnification } M = m \times m_o = -\frac{s d_o}{f_o f_e}$$

As before object or in this case
image of object is placed at
eyepiece focus so rays are parallel
entering eye.

$$f\# = \frac{\text{focal length}}{\text{aperture diam}}$$

not focal length f

Telescope



Object

(focal length F_o)

Focus of objective lens and eyepiece
are made to coincide so again
exitig rays are parallel for eye

$$\theta_0 \approx \frac{h}{F_o}$$

$$\theta_e \approx \frac{h}{F_e}$$

$$m_o = \frac{\theta_e}{\theta_0} = \frac{F_o}{F_e}$$

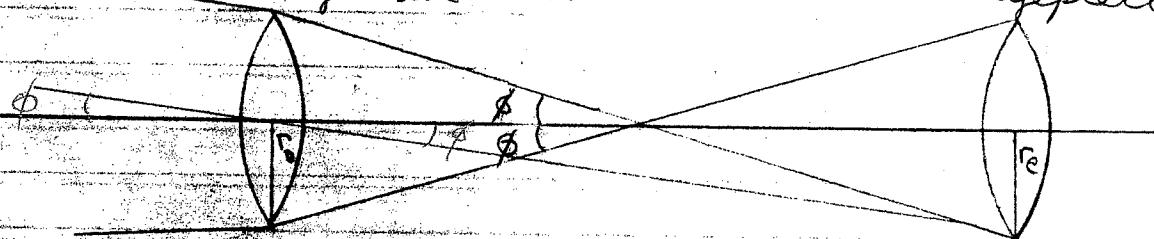
$$\begin{aligned} & \text{Ex } F_o = 1000 \text{ mm} \\ & F_e = 4 \text{ mm} \\ & m_o = 250 \\ & "250 \text{ power} \end{aligned}$$

Telescope Field of View

Determined when rays miss eyepiece

Objective Lens

Eyepiece



$$FOV = 2\phi \quad \text{In the above } h \approx r_e$$

$$\phi = h/F_o = r_e/F_o$$

$$FOV = 2r_e/F_o = d_e/F_o \quad \begin{aligned} & d_e - \text{diameter eyepiece} \\ & = (d_e/d_o)(d_o/F_o) = \frac{1}{f_o} \frac{d_e}{d_o} \quad f_o - F\# \text{ obj} \end{aligned}$$

$$FOV = (F_e/F_o)(d_e/F_e)$$

$$d_o - \text{diam obj}$$

$$= \frac{1}{m_{o,e}}$$

$$f_c - F\# \text{ eyepiece}$$

Limiting magnification of Telescope

$$m_o = F_o / F_e = (F_o / d_o) (d_e / F_e)$$
$$= (F_o / d_o) \frac{1}{f_e}$$

f_e must be ≥ 0.5 (thermodynam

$$\rightarrow m_o < 2 F_o / d_o$$

If $d_e <$ iris diameter of eye image
partially lost (fuzzy)

Iris diam 2 mm bright light
7 mm dark

For normal eyepiece $f_e \approx 1$ (0.5
hard to make well)

Inpired: $m_o \approx F_o / d_o$ $f_e \approx 1$

$m_o \approx d_o / d_e$ $f_o \approx f_e \approx 1$

Common in binoculars