

Polarization

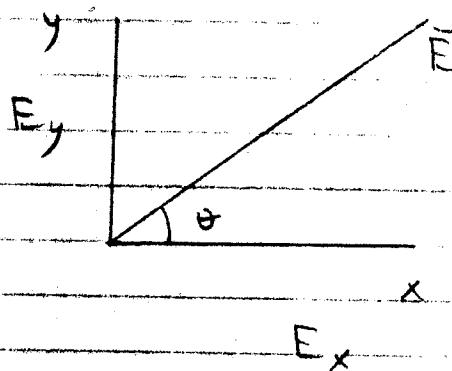
1) Orthogonal vectors cannot interfere

$$\bar{E} = \bar{E}_x + \bar{E}_y$$

$$I \propto E^2 = \bar{E} \cdot \bar{E} = E_x^2 + E_y^2 + 2\bar{E}_x \cdot \bar{E}_y$$

$$= E_x^2 + E_y^2 - \bar{E}_x \cdot \bar{E}_y$$

if no interference



In general $\bar{E} = E_x \hat{E}_x + E_y \hat{E}_y$

$$\tan \theta = E_y / E_x$$

$$E_x = E \cos \theta \quad E_y = E \sin \theta$$

If we use a polarizer sensitive to x direction

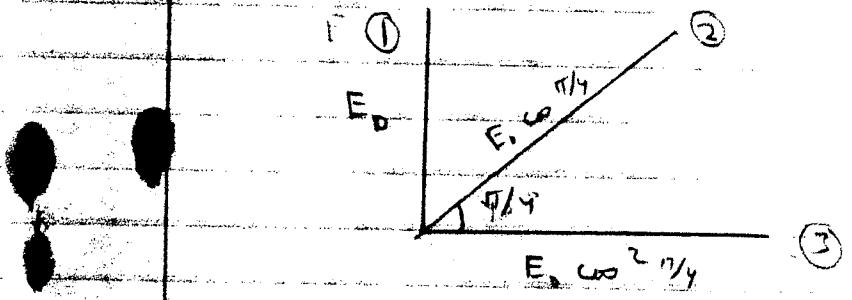
$$I = I_0 \cos^2 \theta$$

$$\text{since } E_x = E \cos \theta$$

at 45° intensity $\approx \frac{1}{2}$

Crossed polarizers - 0 out

Now put another polarizer
at 45° between -
what happens.



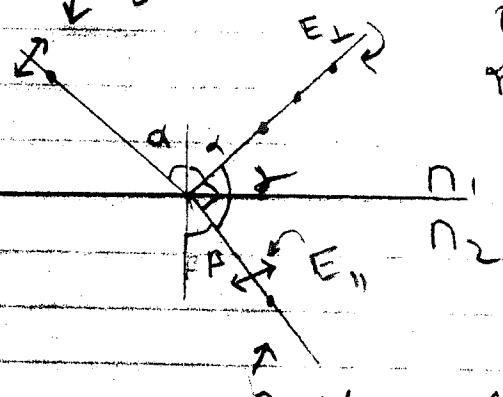
$$I_2 = I_0 \cos^2 \frac{\pi}{4}$$

$$I_3 = I_0 \cos^4 \frac{\pi}{4} = I_0 (\frac{1}{\sqrt{2}})^2 = I_0 / 4$$

Can use polaroid sheet to
rotate polarization - not very good
because of absorption of other polarizers
at each sheet.

Brewster angle

Random polarizations in



Only E_{\perp} out
no E_{\parallel} reflected

air
glass

$E_{\perp} \perp$ to plane sca
 $\Rightarrow E_{\parallel} \parallel$ " "

Both pol out

Brewster angle α such that
 $\alpha + \beta = \pi/2 \rightarrow \beta = \pi/2 - \alpha$

ie reflected & refracted wave
are \perp

$$n_1 \sin \alpha = n_2 \sin \beta = n_2 \sin(\pi/2 - \alpha) \\ = n_2 \cos \alpha$$

$$\theta_B = \alpha = \tan^{-1} \frac{n_2}{n_1}$$

For air-glass

$$n_1 = 1 \quad n_2 = n \sim 1.5$$

$$\theta_B = \tan^{-1} n \sim \underline{\underline{56^\circ}}$$

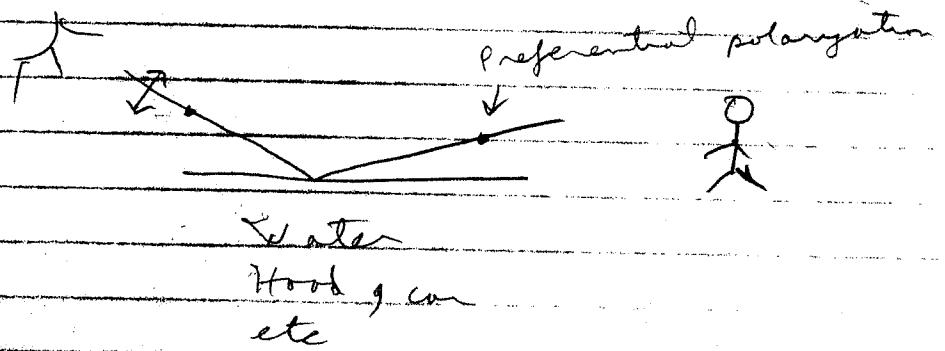
Brewster angle

In by are reflected & refracted
says I for no E_{\parallel} reflected?
Feynman I-23-4

Because we can think of
reflected ray as causing vibrating
electrons in surface. But oscillating
electrons don't radiate along their
direction of motion which would be
the case for E_{\parallel} .

Note E_{\perp} is parallel to surface

Reflected light is preferentially
polarized parallel to surface



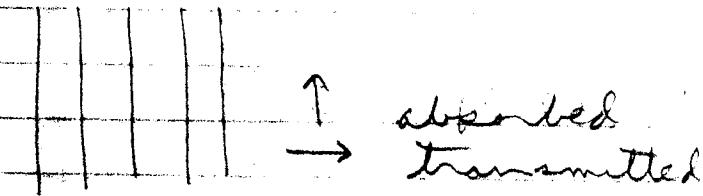
Get polarized light from
unpolarized light in

∴ Surface (glass) can polarized reflected light.

Why do polaroid glasses cut glare off water, car hood?

Teyman T - 22 - 4

Polaroid sheet - crystals of beropatite (iodine & quinine)
Iodine atoms (conductor) on a polymer sheet. Like wires.



If wires are used then rejected radiation is largely reflected not absorbed.

Circular polarization

Take 2 +
π/2 out of
total electric
field look like?

$$\bar{E}_r = \bar{E}_x \cos(kz - wt)$$

$$\bar{E}_z = \bar{E}_y \sin(kz - wt)$$

$$\bar{E} = \bar{E}_r + \bar{E}_z$$

Wave is circularly polarized

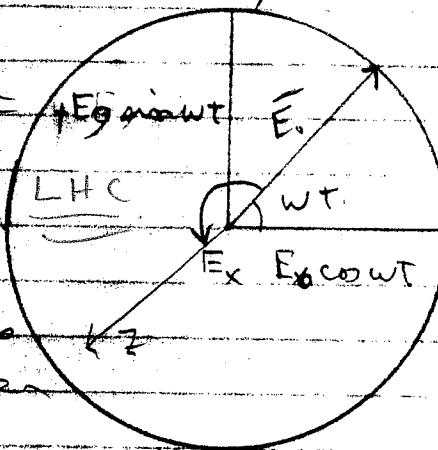
1) Right circular if right hand rule for E field rotating looking along direction of propagation toward source = Clockwise E rotation

2) Left

CCW E rotation

E field rotates at freq $\nu = \frac{w}{2\pi}$

at $z=0$



Light travels kz
out of paper

Looking at source

R H C

$$E_x = \cos(kz - wt)$$

$$E_y = +\sin(kz - wt)$$



L H C

$$E_x = \cos(kz - wt)$$

$$E_y = -\sin(kz - wt)$$



Photons & Spin

Photons can have only two spin states since they travel at c . Spin (angular momentum) states are either along or opposite to direction of motion.

L - Spin (ang mom.) $\pm \hbar$

$$\hbar = h \text{ (Planck's constant)} / 2\pi$$

$$h \approx 6.63 \times 10^{-34} \text{ joule-sec}$$

v - Energy of photon $= h\nu$

ν - ang freq.

$$\rightarrow L = \gamma v - \text{valid for}$$

a classical circularly polarized wave

For total absorptions $L = \gamma v$

For total reflection $L = 2v/\omega$

why

E field at reflector undergoes a π phase shift $\rightarrow E$ changes direction by $\pi \rightarrow$ RCP in goes to LCP out

ang. mom. conservation requires $L \geq 2v/\omega$

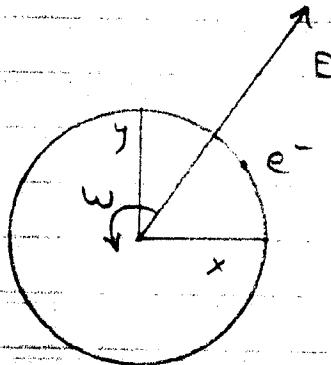
Same as in a mirror a right handed screw looks left handed

In mechanical systems recall

$$L = I\omega$$
$$V = \frac{1}{2}I\omega^2$$

$$L = 2V/\omega$$

Consider circularly pol. light falling on a absorbing atom which we model as a simple harmonic osc. electron



$F_x \propto E_x$ → $F_y \propto E_y$ → electron will tend to rotate in the same sense as pol. of light.

T - torque on electron

P - power delivered to electron

L - ang. mom. delivered

V - energy delivered

$$V = P\tau \quad P = T\omega \quad U = T\omega t$$

$$L = T\tau$$

$$\rightarrow L = \underline{\underline{V/\omega}}$$

Stokes Parameters & Poincaré Sphere

Assume wave along +z axis

\vec{E} is x-y plane

$$E_x = a_1 \cos(\omega t - kz + \delta_1), I = a_1 \cos^2 \delta_1$$

$$E_y = a_2 \cos(\omega t - kz + \delta_2) = a_2 \cos(\tau + \delta_2)$$

$$(1) \quad \tau = \omega t - kz$$

$$\frac{E_x}{a_1} = \cos \tau \cos \delta_1 - \sin \tau \sin \delta_1$$

$$\frac{E_y}{a_2} = \cos \tau \cos \delta_2 - \sin \tau \sin \delta_2$$

$$\Rightarrow \frac{E_x}{a_1} \sin \delta_2 - \frac{E_y}{a_2} \sin \delta_1 = \cos \tau \sin (\delta_2 - \delta_1)$$

$$\frac{E_x}{a_1} \cos \delta_2 - \frac{E_y}{a_2} \cos \delta_1 = \sin \tau \sin (\delta_2 - \delta_1)$$

$$\Rightarrow \left(\frac{E_x}{a_1} \right)^2 + \left(\frac{E_y}{a_2} \right)^2 - 2 \frac{E_x}{a_1} \frac{E_y}{a_2} \cos \delta = \sin^2 \delta$$

$$\delta = \delta_2 - \delta_1 = \text{phase shift}$$

This is the equation of ellipse
since determinant ≥ 0

$$\begin{vmatrix} \frac{1}{a_1} & -\frac{\cos \delta}{a_1 a_2} \\ -\frac{\cos \delta}{a_1 a_2} & \frac{1}{a_2} \end{vmatrix} = \frac{1}{a_1^2 a_2^2} (1 - \cos^2 \delta) = \frac{\sin^2 \delta}{a_1^2 a_2^2} \geq 0$$

Born & Wolf Pg 25

In general wave is elliptically polarized

I Linear pol

$$Y \quad \delta = \delta_2 - \delta_1 = m\pi \quad m \geq 0, \pm 1, \pm 2$$
$$\delta_1 = \delta_2 - m\pi$$

$$E_x = a_1 \cos(\tau + \delta_1) = a_1 \cos(\tau + \delta_2 - m\pi)$$
$$= a_1 \cos(\tau + \delta_2)(-1)^m$$
$$= E_y \frac{a_1}{a_2} (-1)^m$$

$$\frac{E_y}{E_x} = \left(\frac{a_2}{a_1}\right) (-1)^m$$

II

Circular pol.

$$Y \quad a_1 = a_2 = a \quad (\text{Equal ampl})$$
$$\delta = (m \pm \frac{1}{2})\pi \quad m = 0, \pm 1, \pm 2$$

$$E_x^2 + E_y^2 = a^2$$

$$\sin \delta = b \quad RHC$$

$$\sin \delta = -b \quad LHC$$

Stokes Parameters

$$S_0 = a_1^2 + a_2^2 \quad \text{as Intensity} \quad I$$

$$S_1 = a_1 - a_2$$

$$S_2 = 2a_1 a_2 \cos \delta$$

$$S_3 = 2a_1 a_2 \sin \delta \quad V$$

not independent since

$$S_0^2 = S_1^2 + S_2^2 + S_3^2$$

H Linear Pol

$$\delta = m\pi$$

$$\Rightarrow S_3 = 0$$

$S_1, S_2 \neq 0$ in general
 $V = 0, Q, U \neq 0$ in general

H Circular Pol

$$\delta = (m + \frac{1}{2})\pi, a_1 = a_2$$

$$\Rightarrow S_1 = S_2 = 0$$

$$S_3 = 2a^2 \neq 0$$

$$Q = U = 0$$

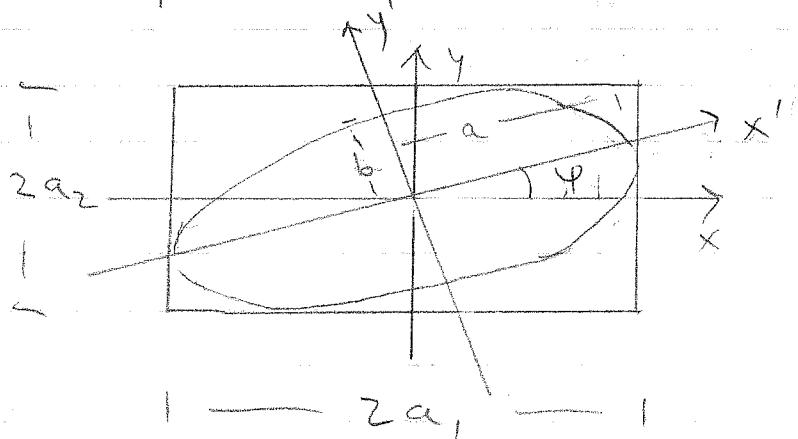
$$V \neq 0$$

$$Q = \frac{I_x - I_y}{2\sqrt{I_x I_y}} \cos S$$

$$U = \frac{I_x - I_y}{2\sqrt{I_x I_y}} \sin S$$

Poincaré Sphere

Rotate general ellipse to principle axes x' , y'



$$E_x = a_1 \cos(\tau + \delta_1)$$

$$E_y = a_2 \cos(\tau + \delta_2)$$

$$E_{x'} = a_1 \cos(\tau + \delta)$$

$$E_{y'} = b_1 \sin(\tau + \delta_0) \quad \leftarrow \text{Note } \underline{\sin}$$

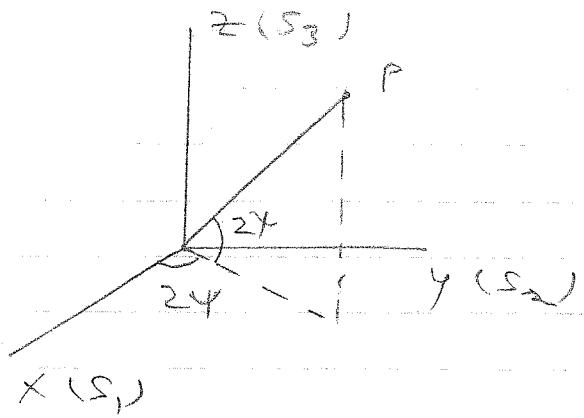
Let $\frac{a_2}{a_1} = \tan \alpha$ (Defined)

Then (can show):

$$\tan 2\varphi = \tan 2\alpha \cos \delta$$

$$\sin 2\chi = -\sin 2\alpha \sin \delta$$

$$a_1^2 + a_2^2 = a^2 + b^2$$



Every polarization state can be represented by a point P on the Poincaré sphere.

In terms of Stokes parameters

$$\begin{aligned} s_1 &= s_0 \cos 2x \cos 2y & x \\ s_2 &= s_0 \cos 2x \sin 2y & y \\ s_3 &= s_0 \sin 2x & z \end{aligned}$$

I Linear pol $\delta = m\pi$
 $\Rightarrow x = 0$

in x, y plane
 \checkmark since $s_3 = 0$

II Circular pol $\delta = (m + \frac{1}{2})\pi$
 $\Rightarrow x = \pi/4$ $a_1 = a_2 = a$
 $\Rightarrow 2x = \pm \pi/2$ $\sin \delta = \pm 1$

R H C $2x = +\pi/2$ North Pole
 $s_3 = s_0$

L H C $2x = -\pi/2$ South Pole
 $s_3 = -s_0$

Optical activity

Many organic structures DNA etc are helical structures ie they have a certain handedness

Light circularly polarized with the same handedness will propagate with a different velocity than light with the opposite handedness (helicity) \rightarrow linear pol. and will rotate plane of pol.

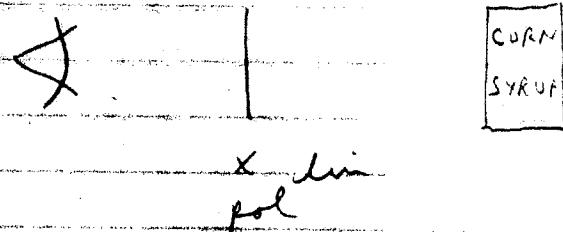
Why?

Examples

Corn syrup
Sugar water
Sugar crystals

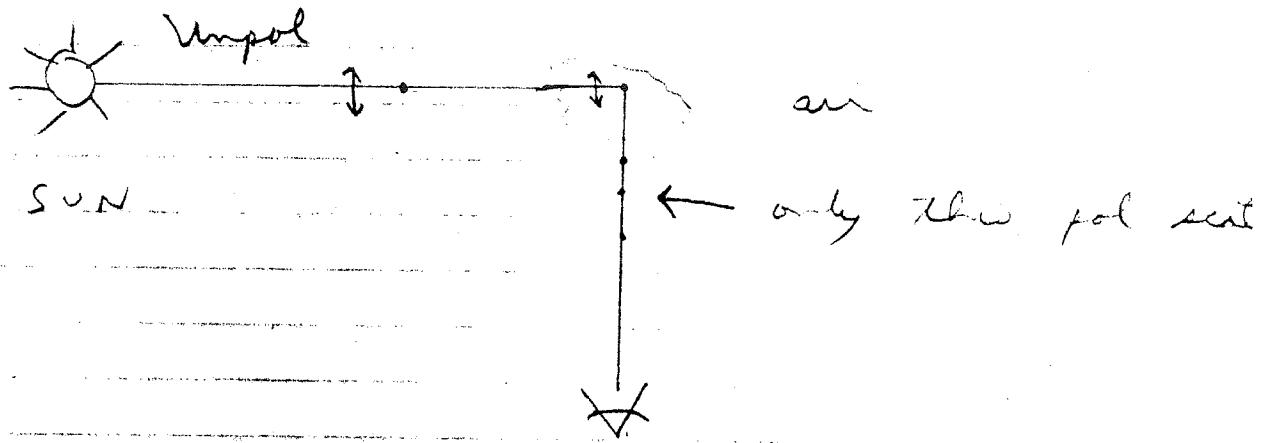
single test

Light in \rightarrow Light out
Light in \rightarrow Light (rot.)



We can describe material as having different indices of refraction n_+ , n_- for different circ. pol.

Scattering of sunlight



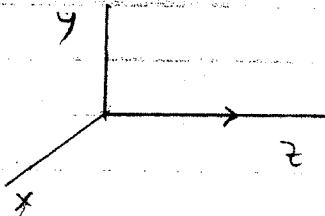
Scattered light is pol \perp to dir
of observation & \perp to source (sun)
why is sky blue.

Birefringence

Different n for different linear polarizations.

Why?

Molecules are "long" and will in general behave differently for different linear polarizations.



↑ "Optic axis"
"long" molecule

E_x, E_y travel at different speeds given by n_x, n_y in material

So what! - useful for getting polarisation and for converting linear circular pol.

Example

1) Quarter wave plate

Let $\Delta n = n_x - n_y$

phase difference for x, y pol. waves

$$\Delta \phi = \frac{l}{\lambda n_x} 2\pi - \frac{l}{\lambda n_y} 2\pi = \frac{2\pi l}{\lambda} \Delta n$$

if $\Delta \phi = \pi/2$ then circle pol.

$l = \frac{\lambda}{4\Delta n}$ Start wave with E at 45°

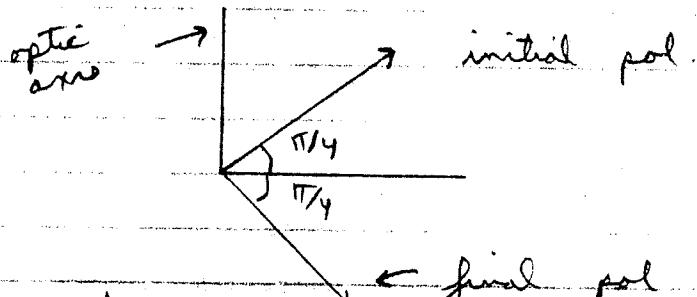
2) Half wave plate

$$\text{if } \Delta\phi = \frac{2\pi l}{\lambda} \Delta n = \pi$$

$$l = \frac{\lambda}{2\Delta n}$$

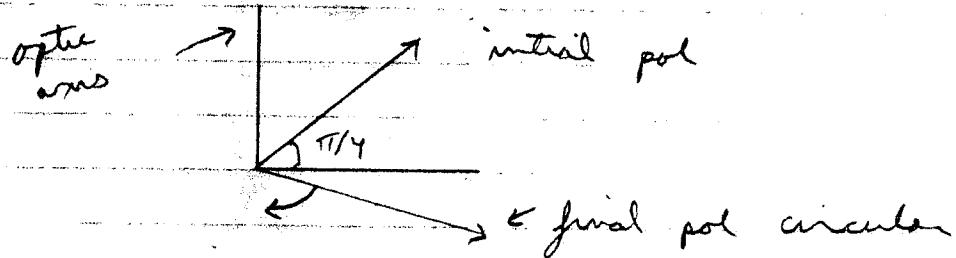
then rad. is lin polarized again.
but rotated by 90° (ie \perp to plane)

assuming as before that initial
radiation is lin pol. at 45°



Lin in (at $\pi/4$) \rightarrow Lin out rot by $\pi/2$
Lin in \rightarrow Lin out (opposite sense)

1) Back to quarter wave plate



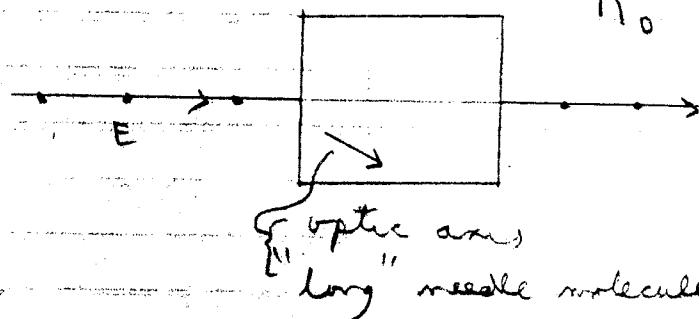
Lin in (at $\pi/4$) \rightarrow cir out
Lin in \rightarrow Lin out

Anisotropic Crystals (Double Refraction)

Figure 1

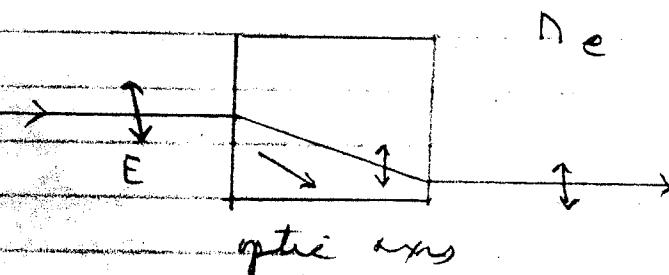
General case of birefringence where "long" molecules "optic axis" can be arbitrarily oriented.

1) $\vec{E} \perp$ optic axis - ordinary ray



E is everywhere \perp to "long" molecule
in ordinary ray

2) Let $\vec{E} \parallel$ optic axis extraordinary.



n_e, n_o
different

Quartz 1.544 1.5
Calcite 1.658 1.4

Part of E is \parallel to "long" molecule (optic axis)
part of E is \perp to "long" molecule (optic axis)
is extraordinary ray.

Note: General case of arbitrary orientation can be very complicated

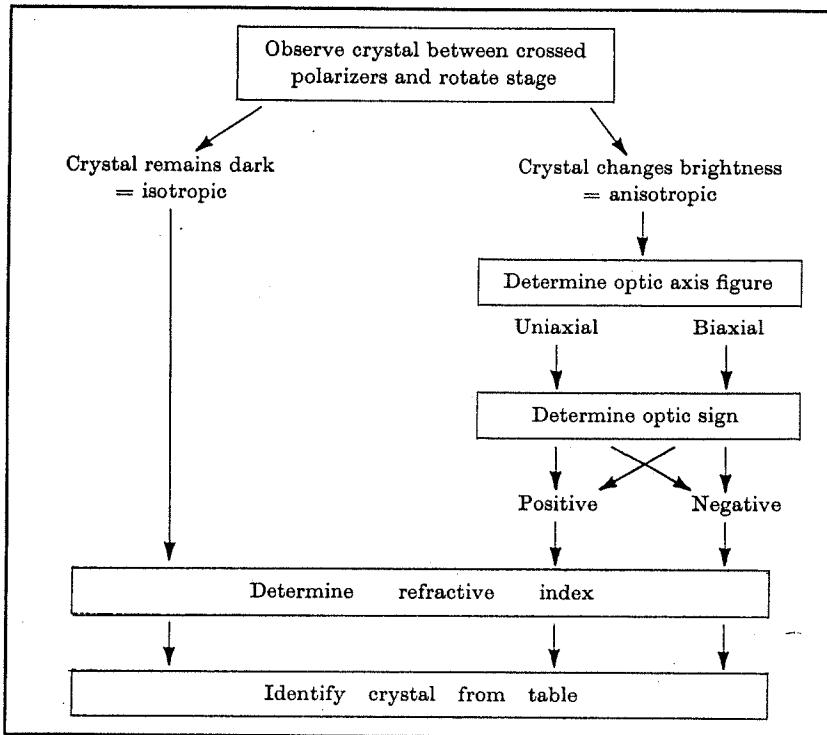
Note : Ordinary ray and extraordinary ray have \perp polariza-

n_o - index of incident direction
 n_e - depends on " "

n_e usually given as
principal index where pol
is \perp to optic axis

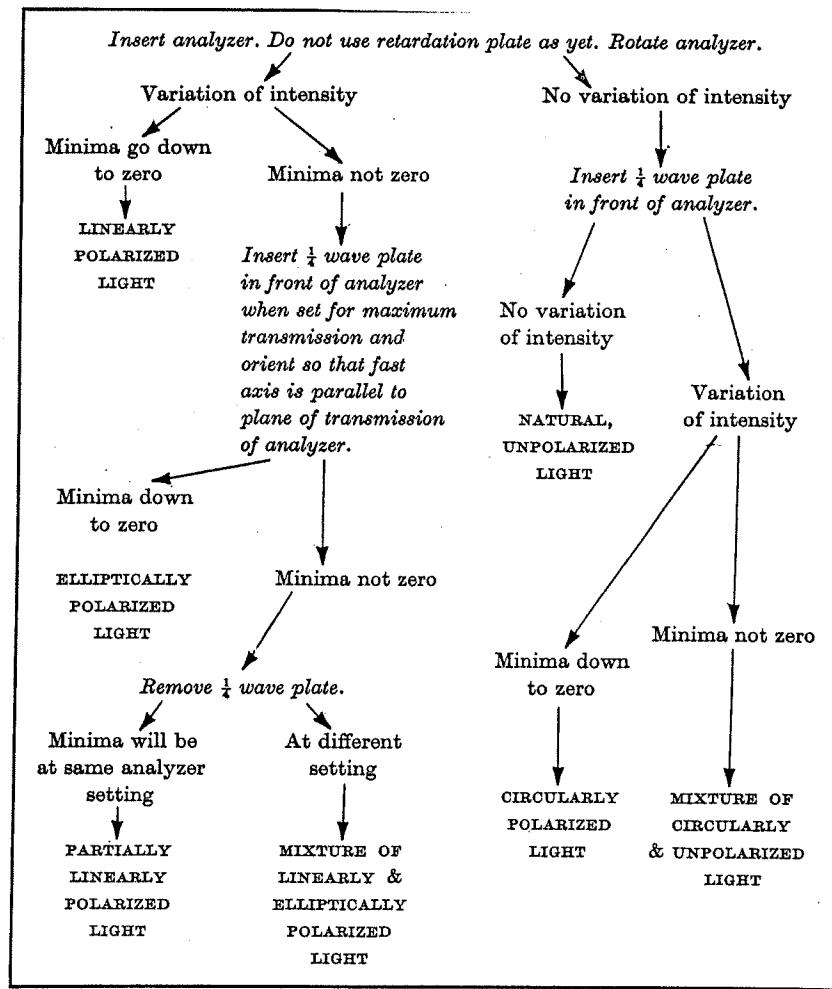
Intro To Classical & Modern Optics

Table 4 OUTLINE OF STEPS FOR IDENTIFYING A CRYSTAL



**Table 5 INDICES OF REFRACTION OF VARIOUS CRYSTALS
(FOR SODIUM D LIGHT, $\lambda = 589 \text{ nm}$)**

<i>A. Isotropic</i>			
$n = 1.5230$	Crown glass		
1.7541	Dense flint		
2.4173	Diamond		
<i>B. Anisotropic uniaxial</i>			
$\omega = 1.6584$	$\epsilon = 1.4864$	Birefringence = -0.172	Calcite
1.5442	1.5533	+0.009	Quartz
1.5874	1.3361	-0.251	Sodium nitrate
<i>C. Anisotropic biaxial</i>			
$\alpha = 1.5601$	$\beta = 1.5936$	$\gamma = 1.5977$	Mica (muscovite)
1.3346	1.5056	1.5061	Potassium nitrate
1.4953	1.5353	1.6046	Tartaric acid



Diffraction

Greens Theorem

Recall: divergence theorem

$$\int \nabla \cdot \bar{F} dV = \int \bar{F} \cdot d\bar{\sigma}$$

let $\bar{F} = U \nabla V - V \nabla U$

U, V scalar, continuous & differentiable

$$\nabla \cdot (U \nabla V) = U \nabla^2 V + \nabla U \cdot \nabla V$$

$$\Rightarrow \nabla \cdot \bar{F} = U \nabla^2 V - V \nabla^2 U$$

$$\Rightarrow \int (U \nabla V - V \nabla U) \cdot d\bar{\sigma} = \int (U \nabla^2 V - V \nabla^2 U) dV$$

Let U, V satisfy wave equation

$$\nabla^2 U - \frac{1}{v^2} \frac{\partial^2 U}{\partial t^2} = 0$$

$$\nabla^2 V - \frac{1}{v^2} \frac{\partial^2 V}{\partial t^2} = 0$$

if U, V are harmonic in time
 $U, V \propto e^{i\omega t}$

$$\nabla^2 U = \frac{1}{v^2} \frac{\partial^2 U}{\partial t^2} = \frac{-\omega^2}{v^2} U$$

$$\nabla^2 V = \frac{-\omega^2}{v^2} V$$

$$\Rightarrow \int (U \nabla V - V \nabla U) \cdot d\bar{\sigma} = 0 \quad \# 1$$

Let V be a spherical wave solution

$$V = V_0 \frac{e^{i(kr - wt)}}{r}$$

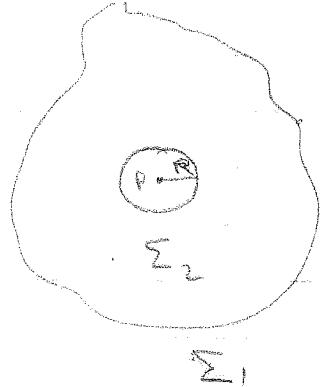
$r=0$ at P

$$\#1 \Rightarrow \int_{\Sigma_1} \left(\frac{e^{ikr}}{r} \nabla V - V \nabla \frac{e^{ikr}}{r} \right) \cdot d\sigma$$

$$- \int_{\Sigma_2} \dots = 0$$

$V_0 e^{-iwt}$ taken out

Σ_2 = sphere centered on P of radius R



W ave V converges to P

let Σ_2 shrink to 0 size $R \rightarrow 0$

$$\lim_{R \rightarrow 0} \int_{\Sigma_2} \frac{e^{ikr}}{r} \nabla V \cdot d\sigma = 0$$

$$\int_{\Sigma_1} V \nabla \left(\frac{e^{ikr}}{r} \right) \cdot d\sigma = \int_{\Sigma_1} V \frac{2}{r^2} \left(\frac{e^{ikr}}{r} \right) R^2 dr$$

$$= \lim_{R \rightarrow 0} \left[- \int_{\Sigma_1} V \frac{e^{ikr}}{r^2} R^2 dr + \int_{\Sigma_1} V k \frac{e^{ikr}}{r^2} R^2 dr \right]$$

$$= - \int_{\Sigma_1} V dr = - 4\pi U_p \quad U_p = V(r=0)$$

$$\Rightarrow U_p = - \frac{1}{4\pi} \int_{\Sigma_1} \left(\frac{e^{ikr}}{r} \nabla V - V \nabla \frac{e^{ikr}}{r} \right) \cdot d\sigma$$

Kirchhoff theorem

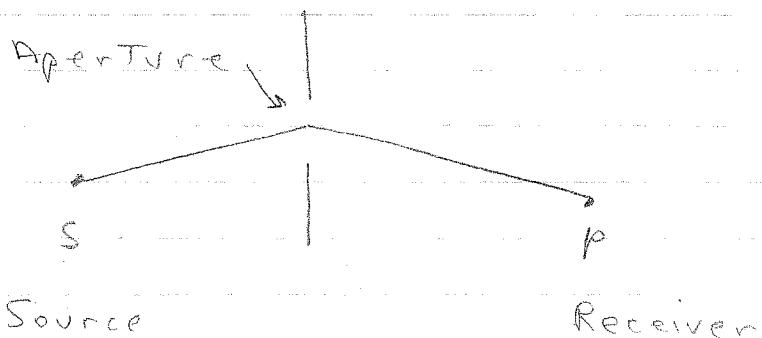
(relates U_{ext} to V on surface Σ)

Since E, B are vectors whereas
 U, V are scalars this is only
an approximation.

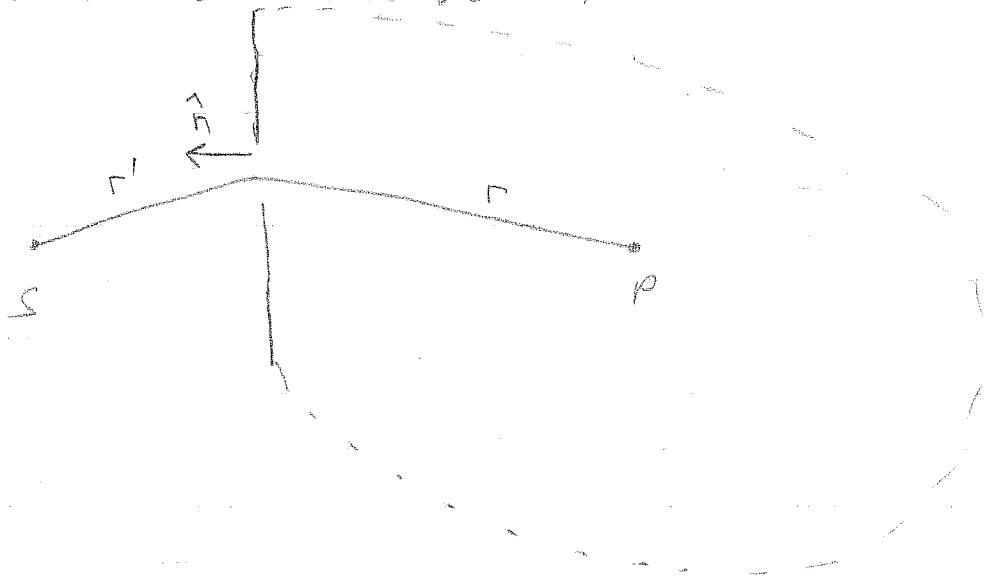
Scalar Diffraction

V = "Optical Disturbance"

Fresnel - Kirchhoff



Now construct a surface Σ_2 that includes the aperture and surrounds P



Make two (extreme?) simplifying assumptions (P on sometimes)

1) $U \propto \nabla U$ contribute negligibly except at aperture

2) $U \propto \nabla U$ are the same at the aperture in absence of partition

If S is a point source
the U at aperture is:

$$U(r') = U_0 \frac{e^{ikr' - wt}}{r'}$$

$$\Rightarrow U_{(sp)} = \frac{U_0 e^{-iwt}}{4\pi} \int_{\Sigma_{Ap}} \left[\frac{e^{ikr}}{r} \nabla \left(\frac{e^{ikr'}}{r'} \right) - \frac{e^{ikr'}}{r'} \nabla \left(\frac{e^{ikr}}{r} \right) \right] d\sigma$$

where Σ_{Ap} = Aperture surface

$$\nabla \left(\frac{e^{ikr}}{r} \right) \cdot d\hat{\sigma} = [e^{ikr} \nabla \left(\frac{1}{r} \right) + \frac{1}{r} \nabla e^{ikr}] \cdot d\hat{\sigma}$$

$$= \left[-\frac{e^{ikr}}{r^2} + \frac{ikr}{r} e^{ikr} \right] \cdot d\hat{\sigma}$$

$$(\nabla \left(\frac{1}{r} \right)) = -\frac{1}{r^2}, \quad \nabla r = -\hat{r}, \quad d\hat{\sigma} = \hat{n}$$

$$= \hat{n} \cdot \hat{r} e^{ikr} \left(\frac{ik}{r} - \frac{1}{r^2} \right)$$

$$\nabla \left(\frac{e^{ikr'}}{r'} \right) \sim \hat{n} \cdot \hat{r}' e^{ikr'} \left(\frac{ik}{r'} - \frac{1}{r'^2} \right)$$

$r, r' \gg$ aperture size

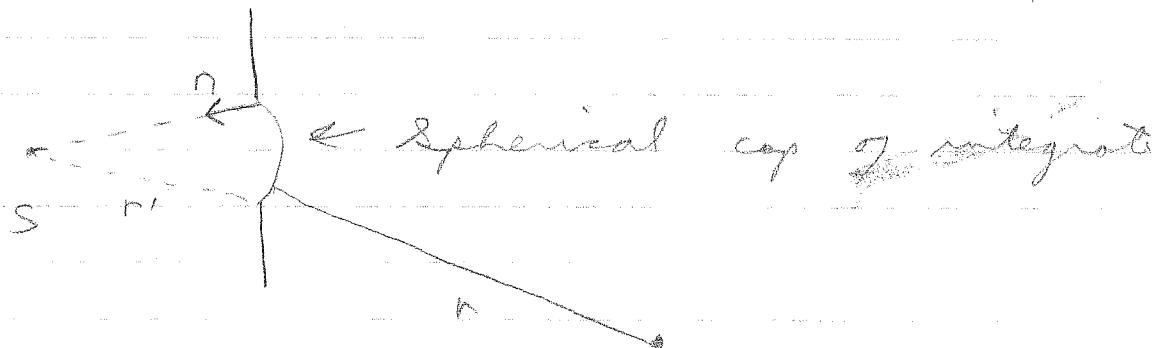
Now assume $r, r' \gg 1$

$$\therefore \frac{k}{r} \sim \frac{1}{\lambda r} \gg \frac{1}{r^2}$$

$$\therefore U_{(sp)} \sim -ikU_0 \frac{e^{-iwt}}{4\pi} \int \frac{e^{ik(r+r')}}{r'r} \cdot (\hat{n} - \hat{n}') \cdot \hat{n} d\sigma$$

$(\hat{n} - \hat{n}') \cdot \hat{n} \equiv$ Obliquity factor
Fresnel - Kirchhoff

Ex : Assume a circular aperture with S symmetrical located



$$\hat{n} \cdot \hat{n}' = -1 \text{ over aperture}$$

$$\Rightarrow U_{AP} = -\frac{ik}{4\pi} \int \frac{U_A e^{ik(r-r')}}{r} (\hat{n} \cdot \hat{n}' + 1) d\sigma$$

$$\text{where } U_A = \frac{U_0 e^{ikr}}{r'}$$

\Rightarrow Incident wave at aperture

This is essentially Huygen's principle

Note in forward direction $\hat{n} \cdot \hat{n}' + 1 = 1$

in backward " $\hat{n} \cdot \hat{n}' + 1 = -1$

$\hat{n} \cdot \hat{n}' + 1 = -1$

so no backward wave

also $-ik \Rightarrow$ displaced wave
is phase shifted by 90°

Babinet's Principle - Complementary

Consider two apertures A_1, A_2

Fraunhofer is linear

$$\Rightarrow \Psi(A) = \Psi_1 + \Psi_2$$

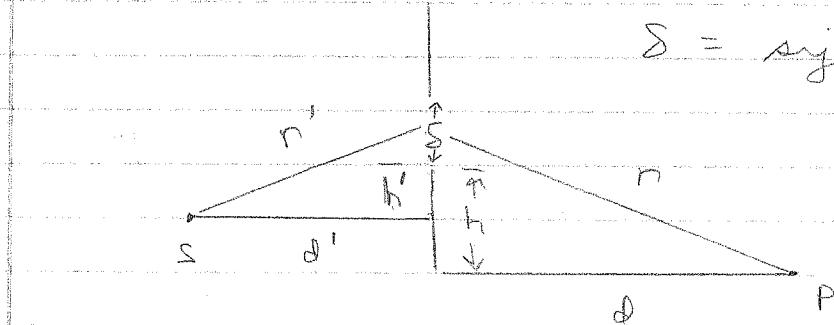
$$\text{Overdue to } A_1 = U_1(\rho) + U_{1s}(\rho)$$

Now if A is a solid opaque screen then $U_{1s} = 0$

$$\Rightarrow U_{1s}(\rho) = -U_2(\rho)$$

π phase shift

Erenbogen & Erenval diants



S = size of aperture

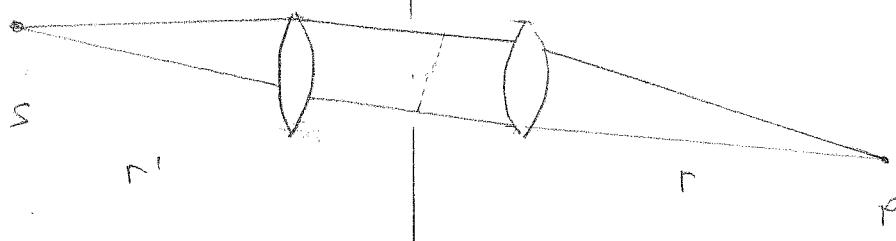
How does $r + r'$ vary over the aperture

Fraunhofer = plane wave at open air

$$\frac{1}{2} \left(\frac{1}{d'} + \frac{1}{d} \right) \delta^2 < 1$$

Otherwise French

Fraunhofer Diffraction



How to make Fraunhofer

Consequences of Fraunhofer assumption

I $\hat{n} \cdot \vec{r} - \hat{n} \cdot \vec{r}' =$ Obliquity factor
does not vary significantly
over aperture
 \Rightarrow Constant in integral

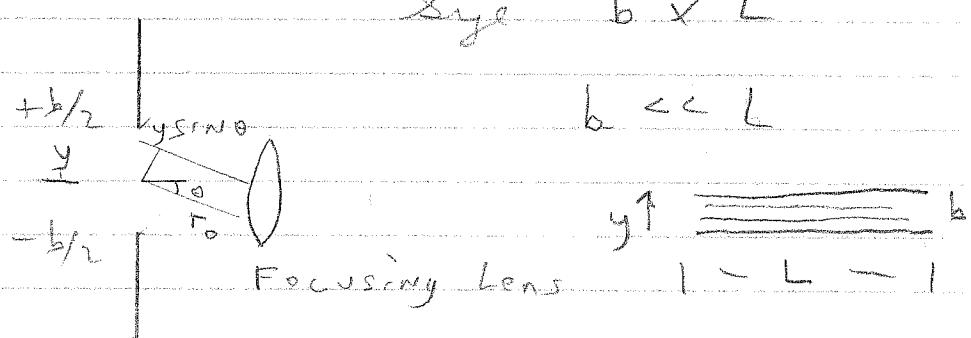
II $V(r)$ is nearly constant
over aperture. (Source is fixed)
 \Rightarrow Constant in integral
 e^{ikr} is varying - if the
plane wave incident is \parallel
to the aperture then e^{ikr}
is also constant

III $V(r)$ is nearly constant
 e^{ikr} varies

$$\Rightarrow V_p (\text{Fraunhofer}) = C \int_{A_p} e^{ik(r+r')} dA$$

Single Slit (Narrow)

Size $b \times L$



$$r = r_0 + y \sin \theta$$

Assume input plane wave is II
to aperture so r' does not
vary

$$U = C e^{ikr_0} \int_{-b/2}^{b/2} e^{iky \sin \theta} dy$$

$$= 2 C e^{ikr_0} L \frac{\sin(k b \sin \theta)}{k \sin \theta} = C' \frac{\sin \beta}{\beta}$$

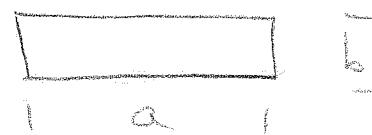
$$\beta = \frac{1}{2} k b \sin \theta \quad C' = e^{ikr_0} C b L$$

$$I \propto |U|^2 \propto I_0 \left(\frac{\sin \beta}{\beta} \right)^2$$

$$\beta = \frac{i 2\pi}{\lambda} b \sin \theta = \frac{\pi b \sin \theta}{\lambda}$$

$$\min \beta = m\pi$$

Rectangular aperture

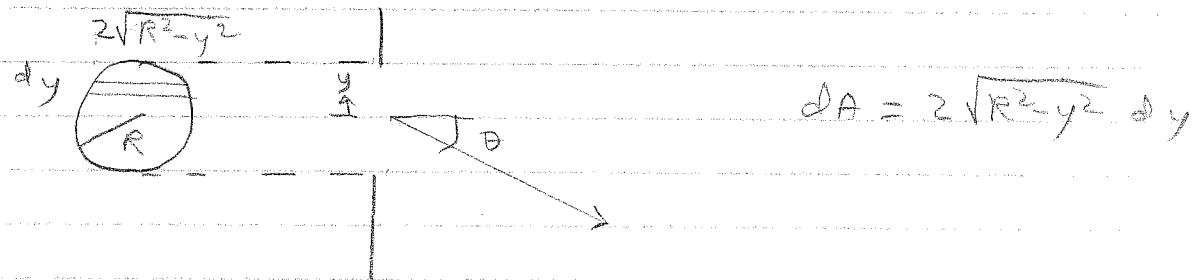


$$I = I_0 \left(\frac{\sin d}{d} \right)^2 \cdot \left(\frac{\sin \beta}{\beta} \right)^2$$

$$d = \frac{1}{2} a \sin \phi$$

$$\beta = \frac{1}{2} b \sin \theta$$

Axial spectrum
Assume incident plane wave II aperture



$$U_p = C e^{ikr_0} \int_{-R}^R e^{iky s \cos \theta} 2\sqrt{R^2 - y^2} dy$$

$$n = y/R, \quad g = KR$$

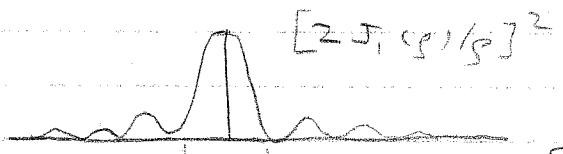
$$\int_{-1}^1 e^{isn} \sqrt{1-n^2} dn = \pi J_1(g)/g$$

J_1 = Bessel function

$$J_1(g)/g \rightarrow 1/2 \quad \text{as } g \rightarrow 0$$

$$\Rightarrow I = I_0 \left[\frac{2J_1(g)}{g} \right]^2$$

$$I_0 = (C \pi R^2)^2$$

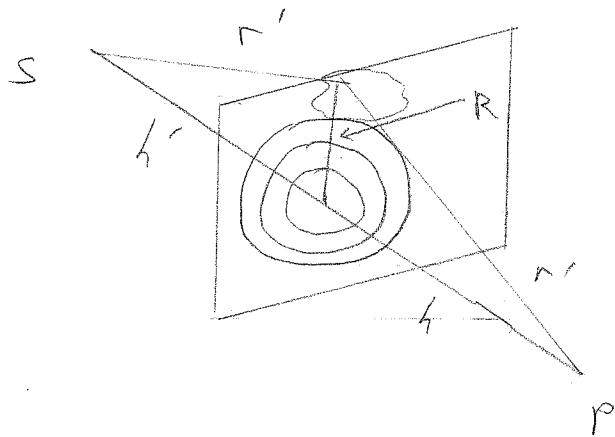


Airy Disk

$$\sin \theta = \frac{3.832}{KR} = \frac{1.22\lambda}{D}$$

$$D = 2R$$

Fresnel Diffraction - zones



$$r + r' = (\hbar^2 + R^2)^{1/2} + (\hbar'^2 + R'^2)^{1/2}$$

$$= \hbar + \hbar' + \frac{R^2}{2} \left(\frac{1}{\hbar} + \frac{1}{\hbar'} \right) + \dots$$

Let's divide into annular zones where $R = \text{constant}$
such that $r + r'$ differ by
 $\lambda/2$

$$R_1 = \sqrt{\lambda L}$$

$$R_2 = \sqrt{2\lambda L}$$

$$R_n = \sqrt{n\lambda L}$$

$$L = \left(\frac{1}{\hbar} + \frac{1}{\hbar'} \right)^{-1}$$

These are Fresnel zones

R_i, R_{i+1} are inner & outer
radii of a zone.

Area of zone

$$A_i = \pi R_{i+1}^2 - \pi R_i^2$$

$$= \pi (i+1) \Delta L - \pi i \Delta L$$
$$= \pi \Delta L = \pi R_i^2$$

all zones have same area

$$V_p = V_1 + V_2 + \dots$$

$$= |V_1| + |V_2| + |V_3| + |V_4| \dots$$

Since zones are the part of plane

Moving

Zone plate

Block alternate Fresnel zones

Let block even ones

$$U_p = |U_1| + |U_3| + |U_5| \dots$$

U_s are adding magnitudes

$$\rightarrow |U_p| > |U_1|, |U_2| \dots$$

P is bright!

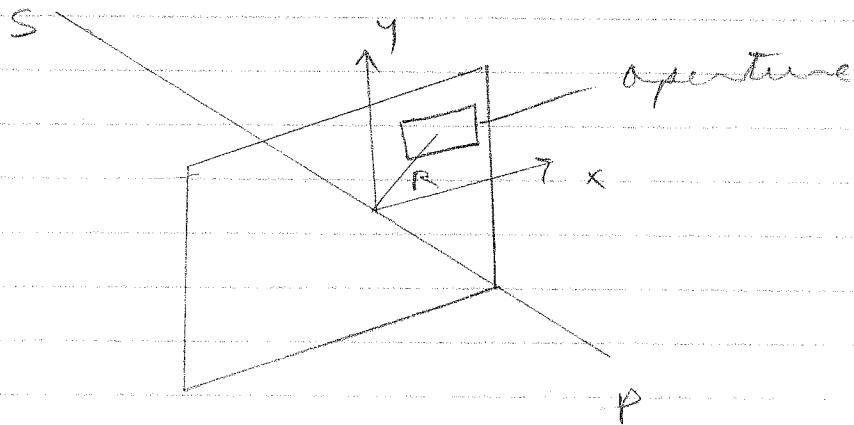
Actually zone plate is a lens

$$\text{Focal length } f = L = R^2/\lambda$$

Examples:

Overhead proj.
Pocket magnifiers
"RV" window stickers

Fresnel Rectangular Aperture



$$R^2 = x^2 + y^2$$

$$r + r' = h + h' + \frac{1}{2}k(x^2 + y^2)$$

$$\frac{\sqrt{x^2 + y^2}}{R}$$

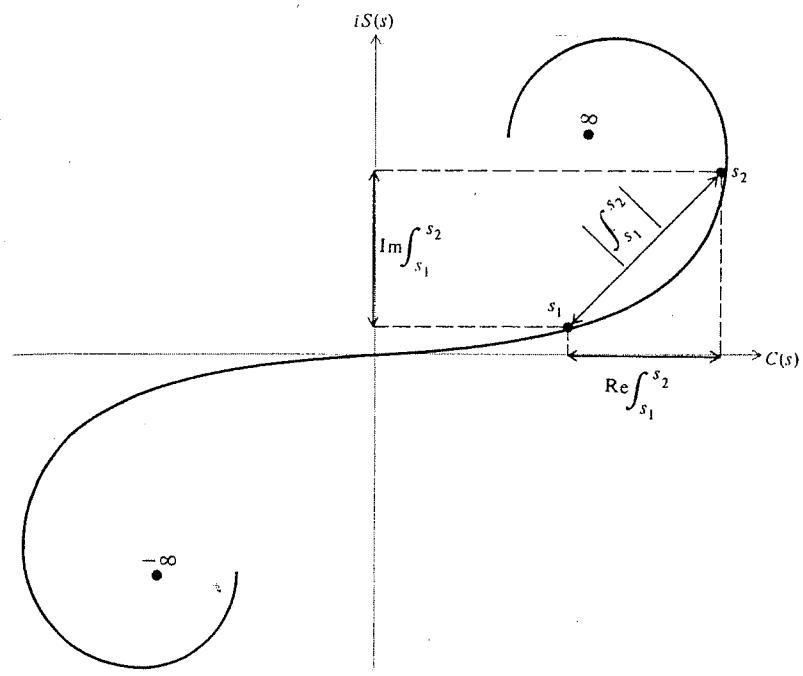
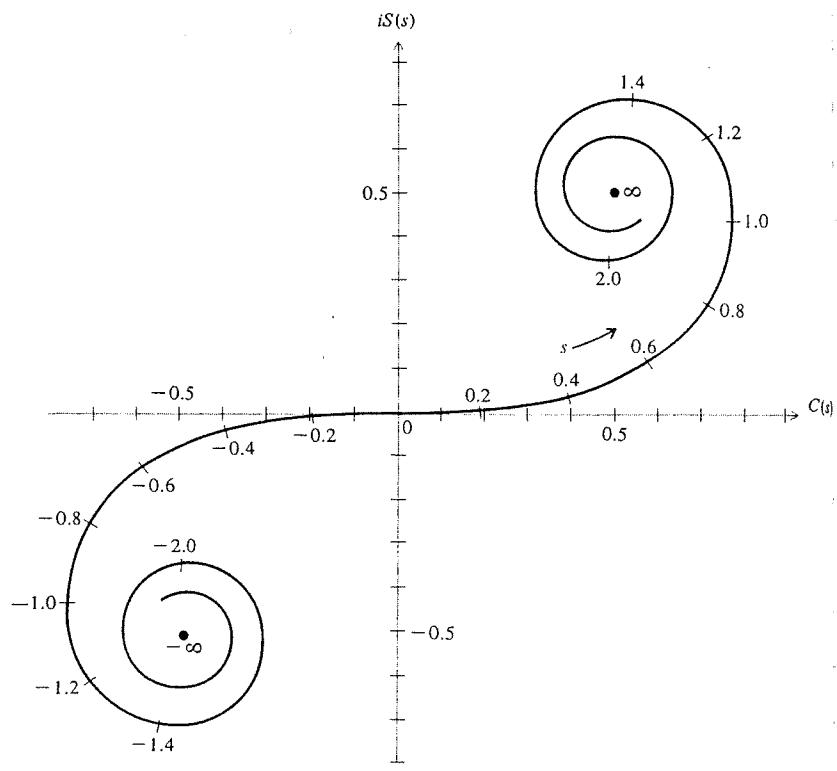
Assume

i) $\hat{r}, \hat{r}' - \hat{h}, \hat{h}'$ waves slowly over aperture \rightarrow Obliquity factor

ii) $U(r')$ waves slowly

$$\Rightarrow U_p = c \int \int e^{ik(x^2 + y^2)/2c} dx dy$$

$$= c \int e^{ikx^2/2c} \int e^{iky^2/2c} dy$$



Define

$$u = x \sqrt{\frac{k}{\pi L}}$$

$$= x \sqrt{\frac{2}{\lambda L}}$$

$$k = \frac{2\pi}{\lambda}$$

$$U_p = U_1 \int_{u_1}^{u_2} e^{i\pi u^2/2} du \int_{v_1}^{v_2} e^{i\pi v^2/2} dv$$

$$U_1 = C\pi L / k$$

$$\int_0^s e^{i\pi w^2/2} dw = C(s) + iS(s)$$

Real part $C(s) = \int_0^s \cos(\pi w^2/2) dw$

Im part $S(s) = \int_0^s \sin(\pi w^2/2) dw$

These are Fresnel Integrals

$C(s)$ & $S(s)$ as called
Cormu spiral

Slit

The rectangle but
 $m_1 = -\infty$, $m_2 = +\infty$

$$U_p = \frac{U_0}{1+i} [c(s) + i s(s)]^{\frac{1}{2}}$$

Straightedge

Slit but with $V_1 = -\infty$

$$U_p = \frac{U_0}{1+i} [c(s) + i s(s)]_{-\infty}^{\frac{1}{2}}$$

$$= \frac{U_0}{1+i} [c(v_2) + i s(v_2) + \frac{1}{2}(1+i)]$$

