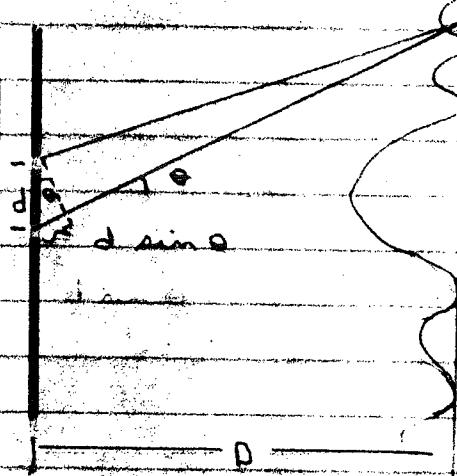


Interference and diffraction

The Huygen's construction at each hole.



assume $D \gg d \rightarrow$ parallel rays

- 1) Constructive interference for integral number of wavelengths

$$d \sin \theta = m \lambda \quad -\text{for maximum}$$

$m = 0, 1, 2, \dots$

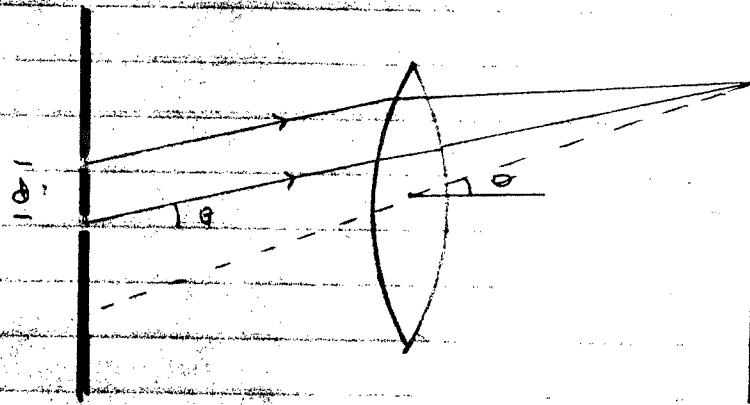
for $m=1 \quad d \sin \theta \approx \theta \quad \theta \ll 1$

- 2) Destructive interference for integral + 1/2 number of wavelengths

$$d \sin \theta = (m + \frac{1}{2}) \lambda \quad -\text{for minimum}$$

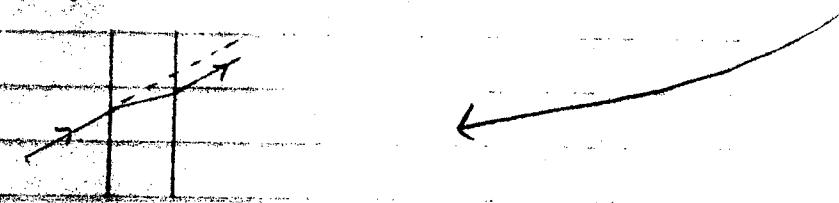
$m = 0, 1, 2, \dots$

What happens when a lens is added



Least Action \rightarrow Least time \rightarrow total time (total phase) for each path is the same

Interference pattern is still the same where $θ$ is measured relative to lens center. But the center is not shifted in angle only a positive first order since for a thin lens surfaces are parallel near center.



Addition of waves

$$\text{let } E_1 = E_0 \sin \omega t \\ E_2 = E_0 \sin (\omega t + \phi)$$

$$E_1 + E_2 = E_0 (\sin(\omega t) + \sin(\omega t + \phi))$$

$$\sin(a+b) = \sin a \cos b + \cos a \sin b$$

$$\sin(a-b) = \sin a \cos b - \cos a \sin b$$

$$\sin(a+b) + \sin(a-b) = 2 \sin a \cos b$$

$$\text{let } a = \frac{A+B}{2} \quad b = \frac{A-B}{2}$$

$$a+b = A \quad a-b = B$$

$$\rightarrow \sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\therefore E = E_1 + E_2 = E_0 \cdot 2 \sin(\omega t + \phi/2) \cos \phi/2$$

Projecting vector

$$\vec{S} = \frac{1}{c\mu_0} \vec{E} \times \vec{B} \quad (|\vec{E}| = |\vec{B}|c)$$

$$\vec{S} = \frac{1}{c\mu_0} E^2 \quad S \propto E^2 \quad (S \text{ intensity})$$

Call intensity \tilde{I} watts/m² (S)

$$\tilde{I} \propto E^2 = 4E_0^2 \sin^2(\omega t + \phi/2) \cos^2 \phi/2$$

In this we see as the true average $\langle \tilde{I} \rangle$

$$\text{Recall } \langle \sin^2 \omega t \rangle = \frac{1}{T} \int_0^T \sin^2 \omega t dt$$

$$\sin^2 \omega t = \frac{1}{2}(1 - \cos 2\omega t)$$

$$\langle \sin^2 \omega t \rangle = \frac{1}{2} \quad \text{if } T \text{ is a period}$$

$$\rightarrow \langle \tilde{I} \rangle = \frac{1}{c\mu_0} 2 E_0^2 \cos^2 \phi/2 \equiv I(\phi)$$

$$\langle I_m \rangle = \frac{1}{c\mu_0} E_0^2 \quad \text{maximum } \phi = 0$$

$$\rightarrow \frac{\langle \tilde{I} \rangle}{\langle I_m \rangle} = \cos^2 \frac{\phi}{\lambda} = \frac{I(\phi)}{I_m}$$

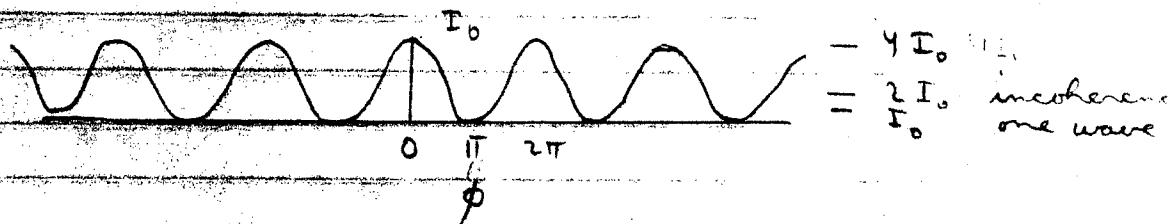
In our two slit problem
 $\phi = \frac{d \sin \theta}{\lambda} \times 2\pi$

$$I(\phi) = I_m \cos^2 \left(\frac{\pi d \sin \theta}{\lambda} \right)$$

In one wave $I_0 = \frac{1}{c\mu_0} E_0^2$
 $\langle \tilde{I}_0 \rangle = \frac{1}{c\mu_0} E_0^2 \frac{1}{2}$

$\therefore I_m = 4 I_0$ not $2 I_0$

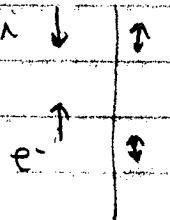
$$I(\phi) = 4 I_0 \cos^2 \left(\frac{\pi d \sin \theta}{\lambda} \right)$$



Coherence

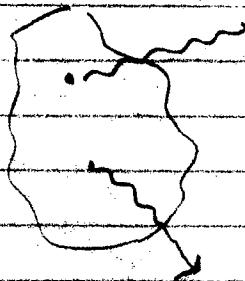
How is light emitted microscopically?

- 1) Accelerated electrons in hot metal
- incandescent light



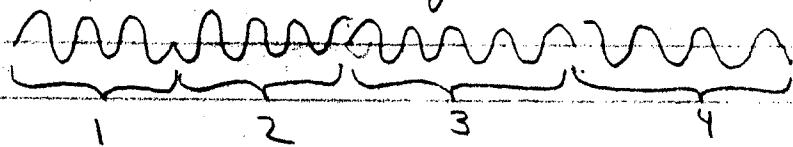
Vibrating electrons at different parts emit randomly phased radiation. Such radiation is incoherent. Each vibrating electron emits a wave for a short time & until it is interrupted by collision.
 τ is the coherence time

- 2) Atoms in an excited gas such as neon



Each atom in an excited state emits a wavetrain for a time & until it is interrupted by collision.
 τ is coherence time
Radiation is incoherent.

Wave form

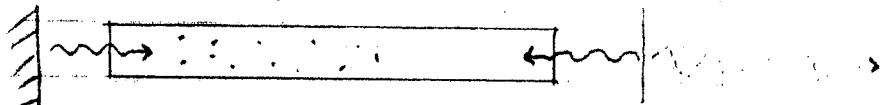


Correction to last lecture

$$\Delta\phi = m \times 2\pi \text{ for maxima}$$

$$\underline{\text{not}} \quad \underline{\Delta\phi} = m\lambda$$

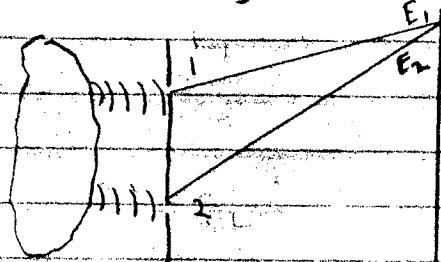
A laser is an example of a coherent radiation source.
Long coherence time.



Atoms are deexcited in unison giving rise to coherent radiation



Testing coherence $\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$



$$E = E_1 + E_2 \quad S = \frac{1}{c\mu_0} E^2$$

$$E_1 = E_0 \sin(\omega t)$$

$$E_2 = E_0 \sin(\omega t + \phi(t))$$

$$E = E_1 + E_2 = 2 E_0 \sin(\omega t + \phi/2) \cos$$

$$I = \frac{1}{c\mu_0} E^2 = \frac{1}{c\mu_0} 4 E_0^2 \sin^2(\omega t + \phi/2) \cos^2$$

but $\sin^2(x) = \frac{1}{2}(1 - \cos 2x)$ $\cos^2(x) = \frac{1}{2}(1 + \cos 2x)$
source $\rightarrow \langle I \rangle = \frac{1}{c\mu_0} E_0^2 = 2 I_0$ (I_0 - for one

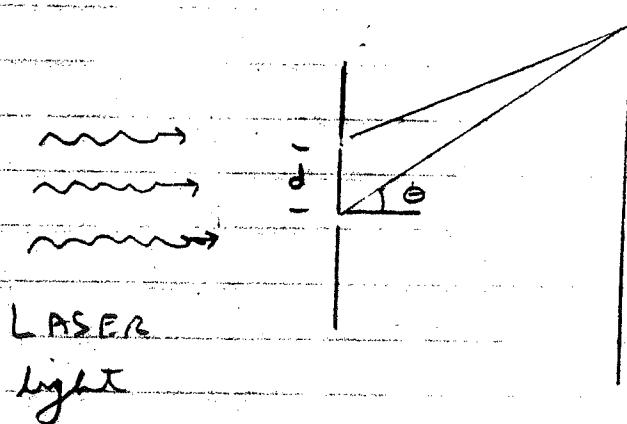
No interference since phases are randomly related

$$I \propto (E_1 + E_2)^2 = 4 E_0^2 \sin^2(\omega t + \phi/2) \cos^2$$

but here $\phi = \phi(t)$ varies

randomly since different parts of surface emit randomly relative to each other.

? intensity is just sum of intensities from slits

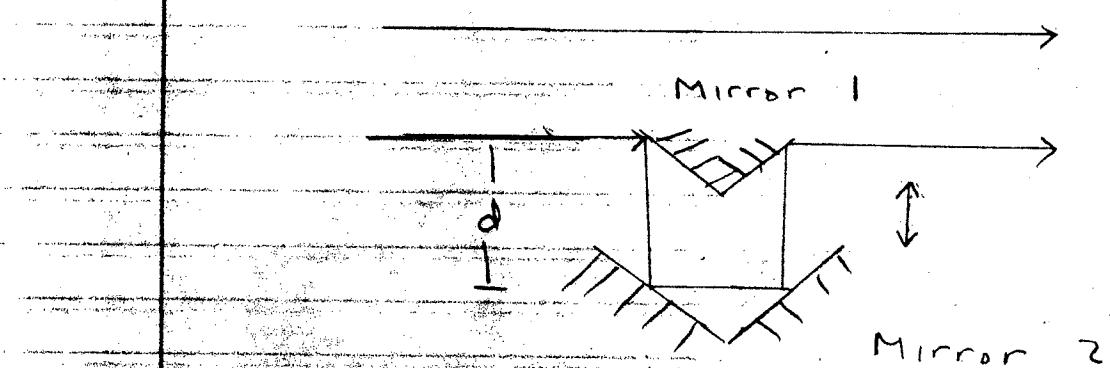


Here we have interference

$$I = 4 I_0 \cos^2 \frac{\phi}{2}$$

$$\text{as before } \phi = \frac{2\pi d \sin \theta}{\lambda}$$

to test of coherence time



Move mirror 2 down until interference fringes disappear. Coherence time τ is then

$$\tau \approx \frac{2d}{c}$$

Misc

v) Wavelength is reduced in dielectric

$$v\lambda_0 = c \quad \text{in vac}$$

$$v\lambda_n = c/n \quad \text{in dielectric}$$
$$\therefore \lambda_n = \lambda_0/n$$

v) Optical path length $\equiv n l$
where l is physical length in
dielectric

why is it useful

$$\Delta\phi = 2\pi \frac{l}{\lambda_0/n} = 2\pi n l \quad \text{in dielectric}$$

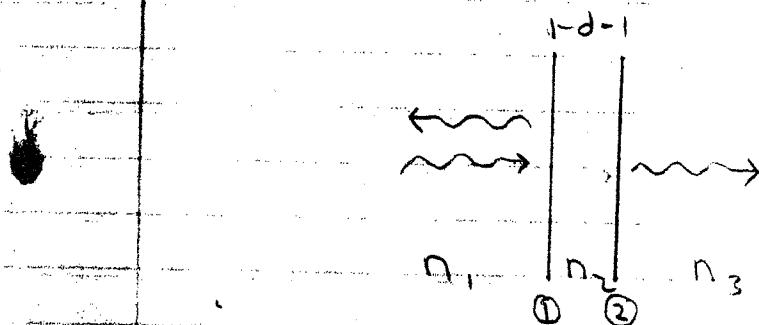
Define $S = n l$

$\Gamma = \Delta S$ path difference

$$\Delta\phi = 2\pi \Gamma / \lambda_0$$

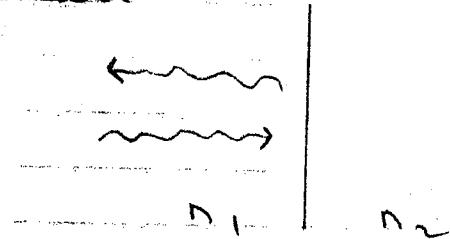
Antireflection Coatings

Reflection from an interface



Assume $n_1 < n_2 < n_3$

1) Reflection at n_1, n_2 boundary - assume infinite second medium



reflected power $r = \frac{(n_1 - n_2)}{(n_1 + n_2)}^2$ (minus)

if $n_2 > n_1$

if $n_1 > n_2$

π phase change on reflect
0 phase " "

2) We want reflection from surface ② to cancel reflection from surface ①.

Since $n_1 < n_2 < n_3$ then π phase change on reflection on both surfaces ① & ②

\therefore Want optical path length
($2nd$) to be a half integral
number of wavelengths for destructive int

$$\frac{2d}{\lambda/n_2} = m + \frac{1}{2} \quad m = 0, 1, \dots$$

$$2nd = (m + \frac{1}{2}) \lambda$$

$$d = \frac{1}{2}(m + \frac{1}{2}) \frac{\lambda}{n_2}$$

3) Reflected amplitudes from ① &
must be equal for complete cancellation
assuming only small reflection \rightarrow

$$\left(\frac{n_1 - n_2}{n_1 + n_2} \right)^2 = \left(\frac{n_2 - n_3}{n_2 + n_3} \right)^2$$

$$\frac{n_1 - n_2}{n_1 + n_2} = \frac{n_2 - n_3}{n_2 + n_3}$$

$$\rightarrow n_1 n_2 + n_1 n_3 - n_2^2 = n_2 n_3 = n_1 n_2 - n_1 n_3 + n_2^2 - n_3^2$$

$$\rightarrow n_2^2 = n_1 n_3$$

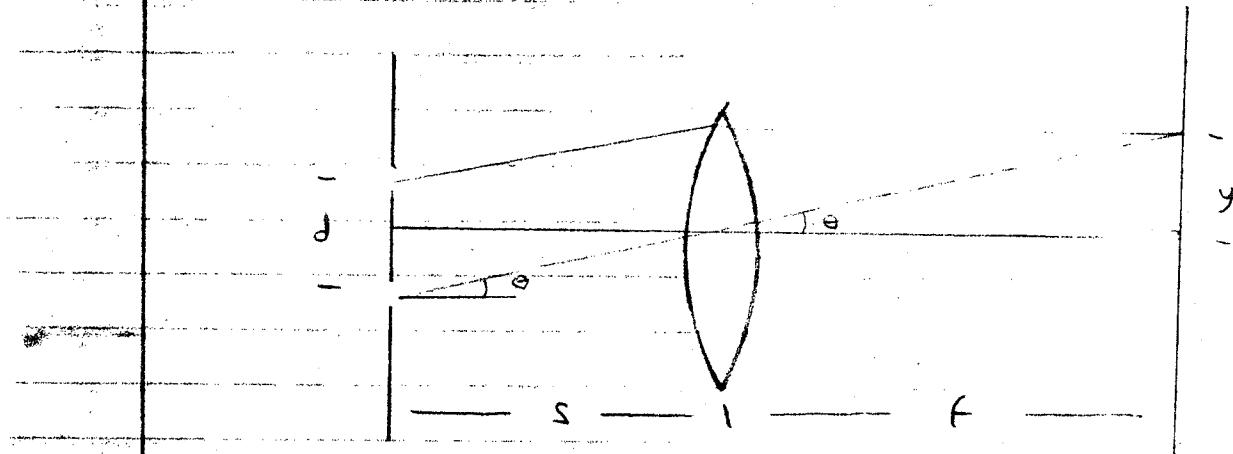
$$\therefore n_2 = \sqrt{n_1 n_3} \quad \text{geometric mean}$$

$$d = \frac{1}{2}(m + \frac{1}{2}) \frac{\lambda}{n_2}$$

$$\text{for } m=0 \quad d = \frac{1}{4} \frac{\lambda}{n_2}$$

Coat lenses with $Mg F$
1/4 reduced wavelength

Lens in interference systems



Screen at T

$d \sin \theta = m\lambda$ for maxima Focus

$$\sin \theta = \frac{m\lambda}{d} = \frac{y}{\sqrt{y^2 + f^2}}$$

$$m^2 \lambda^2 (y^2 + f^2) = d^2 y^2$$

$$y = \frac{m \lambda f}{\sqrt{d^2 - m^2 \lambda^2}}$$

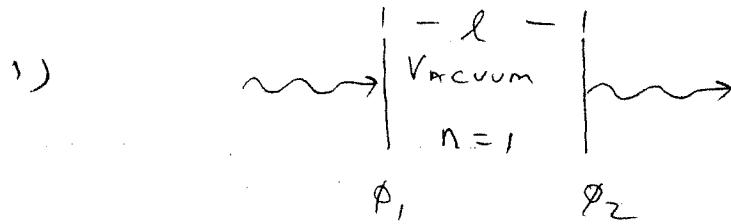
→ i.e. y is independent of s

Interference pattern is independent of position of lens from slits as long as screen is at the lens focus.

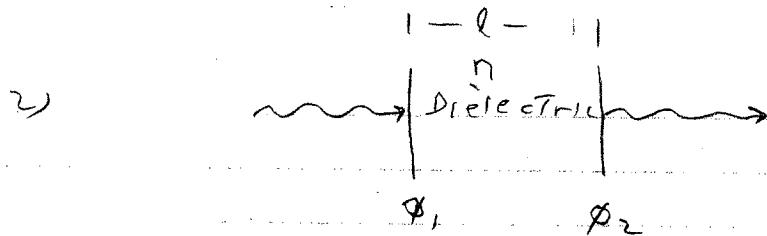
Obviously the pattern depends on the focal length of the lens.

→ Without a lens, pattern distance y depends on distance of screen from slits.

Optical Path Length & Phase Change



$$\Delta\phi = \phi_2 - \phi_1 = 2\pi \frac{l}{\lambda_0}$$

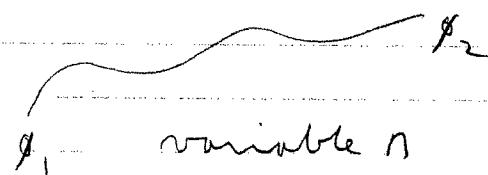


$$\Delta\phi = \phi_2 - \phi_1 = 2\pi \frac{l}{\lambda_n} = 2\pi \frac{l}{\lambda_0/n} = 2\pi \frac{nl}{\lambda_0}$$

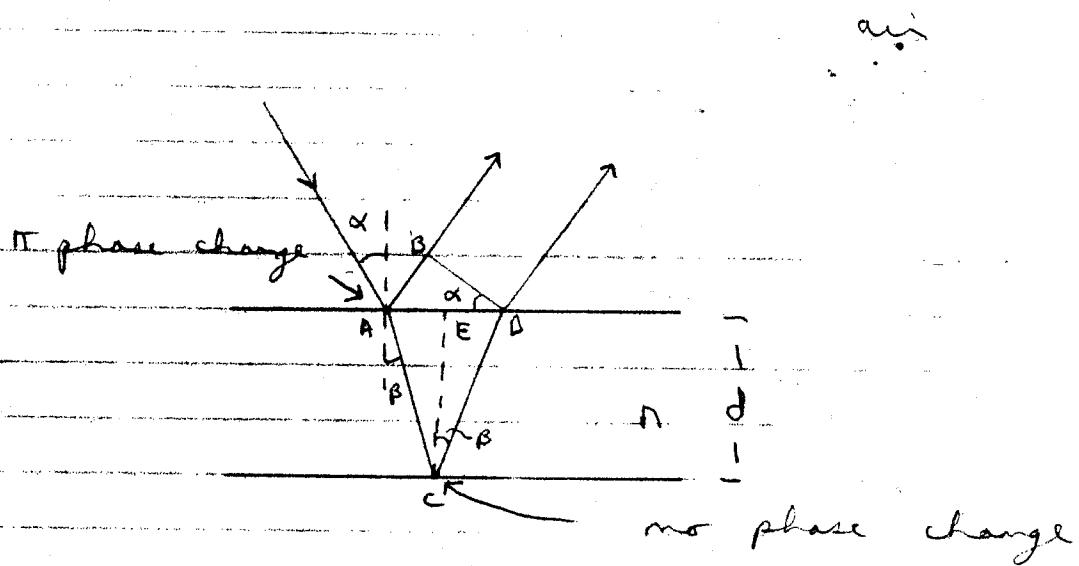
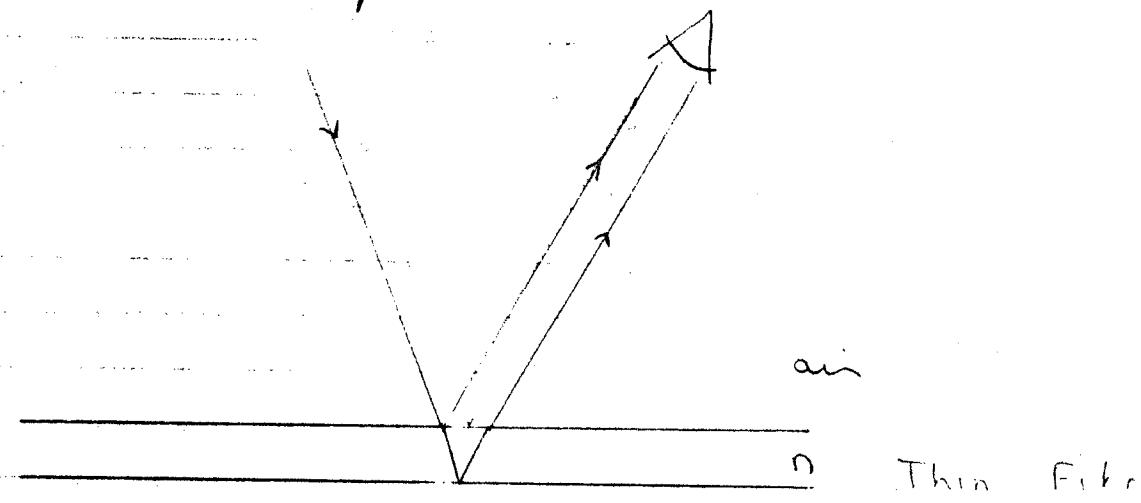
Define $\Gamma = nl$ $\Delta\phi = 2\pi\Gamma/\lambda_0$

3) More generally $\Gamma = \int_C n dl$

$$\Delta\phi = \frac{2\pi}{\lambda_0} \int_C n dl = \frac{2\pi}{\lambda_0} \Gamma = \phi_2 - \phi_1$$



Interference from Thin Films Soap Film



Snell's law $\rightarrow \sin \alpha = n \sin \beta$

$$AC = CD = \frac{d}{\cos \beta} \quad AE = d \tan \beta = \frac{AD}{2}$$

$$\sin \alpha = \frac{AB}{2AE}$$

Γ - Excess Optical Path

$$\begin{aligned} \Gamma &= n(AC + CD) - AB = 2nAC - 2AE \sin \beta \\ &= 2nd/\cos \beta - 2d \tan \beta \sin \beta \\ &= 2nd/\cos \beta - 2d \tan \beta n \sin \beta \quad (\text{Simpl}) \\ &= 2nd(1 - \sin^2 \beta)/\cos \beta = 2nd \cos \beta \end{aligned}$$

$\Gamma = 2nd \cos \alpha$ = optical path length difference between AB and A' C' D

But at first surface there is a $\pi(\frac{1}{2})$ phase shift

$$\cos \alpha = \sqrt{1 - \sin^2 \alpha} = \sqrt{1 - \sin^2 \alpha / n^2} = \frac{\sqrt{n^2 - \sin^2 \alpha}}{n}$$

i.e. ① Interference maxima occur,

$$\Gamma + \frac{\lambda}{2} = m\lambda \quad m = 0, 1, 2, \dots$$

$$\rightarrow 2nd \cos \alpha = (m - \frac{1}{2})\lambda$$

$$2d \sqrt{n^2 - \sin^2 \alpha} = (m - \frac{1}{2})\lambda$$

$$2dn = (m - \frac{1}{2})\lambda \quad \alpha = 0$$

② minima for

$$\Gamma + \frac{\lambda}{2} = (m + \frac{1}{2})\lambda \quad m = 0, 1, \dots$$

$$\rightarrow 2nd \cos \alpha = m\lambda$$

$$2d \sqrt{n^2 - \sin^2 \alpha} = m\lambda$$

$$2dn = m\lambda \quad \alpha = 0$$

Thickness (nm) Max $n = 1.5$ $\alpha = 0$
 (Glass)

40 Blackish

55 Gray

100 White

130 Yellow

150 Brown

175 Red

190 Violet

210 Blue

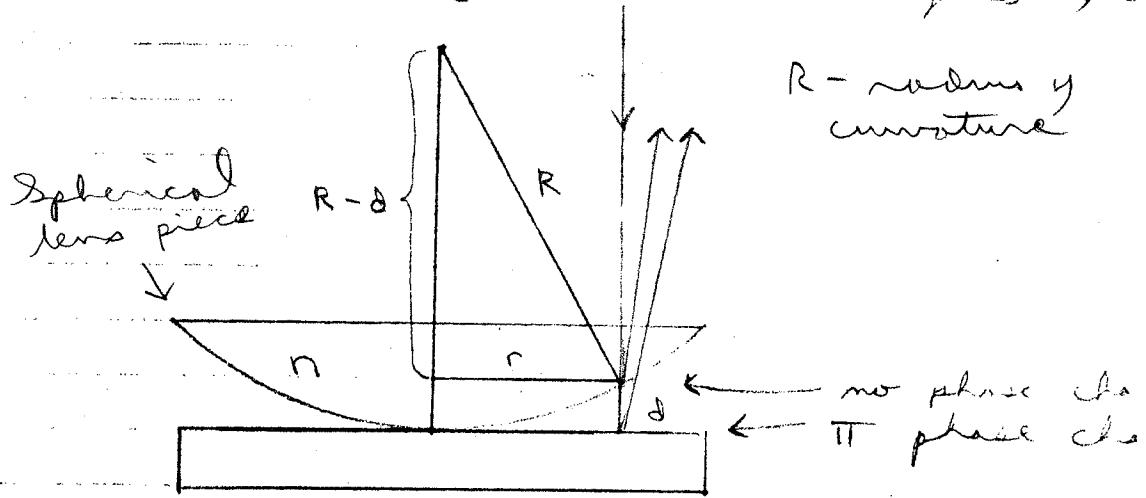
240 Green

260 Yellow-Green

Examples - soap bubble, oil on water

Newton's Rings

Intro to Class & Modern Physics Pg 2



$$R^2 = (R - \delta)^2 + r^2 = R^2 - 2R\delta + \delta^2 + r^2$$

$$\delta^2 - 2R\delta + r^2 = 0$$

$$\delta \approx \frac{r^2}{2R} \quad \text{for } \delta \ll R \rightarrow \delta^2 \approx 0$$

$$\rightarrow r = \sqrt{2R\delta}$$

Use thin film conditions for
min & max since there is still
a π phase change at one surface
 $\propto (m - \frac{1}{2})\lambda$

$$2d = (m + \frac{1}{2})\lambda \quad \text{max}$$

$$2d = m\lambda \quad \text{min}$$

$$r = \sqrt{R}2d$$

$$\Rightarrow \begin{cases} r = \sqrt{R(m + \frac{1}{2})}\lambda \\ r = \sqrt{Rm}\lambda \end{cases} \quad \begin{matrix} \text{max} \\ \text{min} \end{matrix}$$

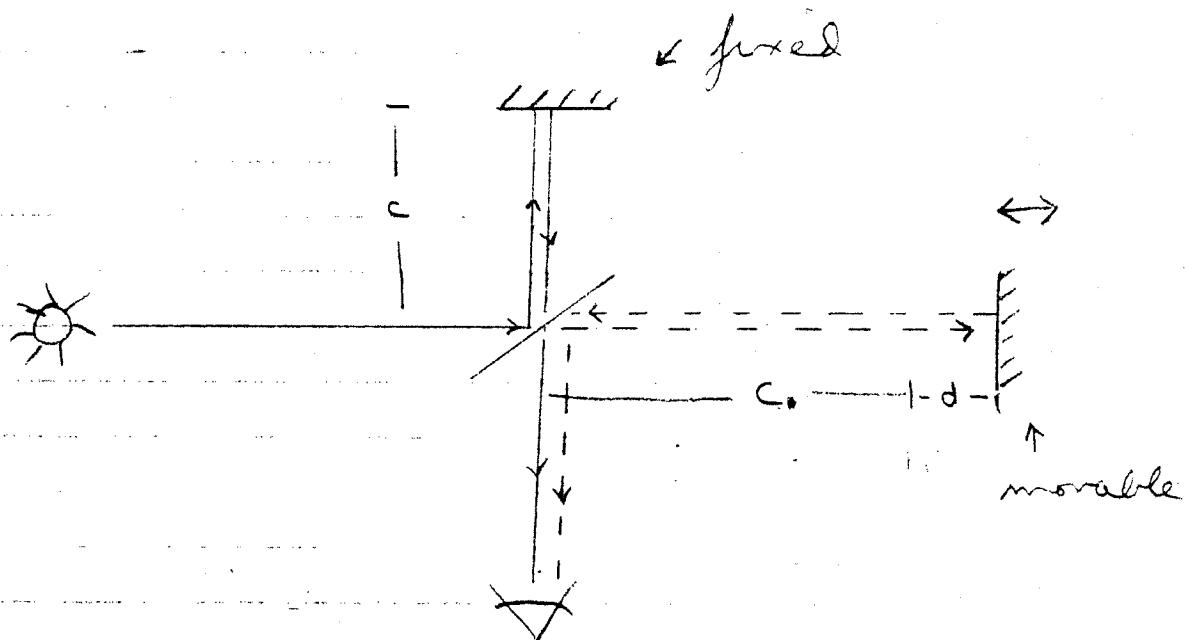
Note for $m=0$ $r=0$ is min
ie center point is dark

Why? - This depends on air gap for π phase change. If inter glass to glass contact center is not

dark . Try some grease !

Interferometers

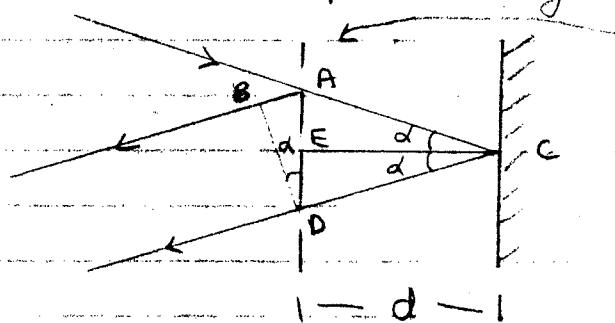
1) Michelson



d = difference in mirror distances

Fringes are circular

Excess path length



Minor would be $\lambda/2$ if arms were equal
(ie $d=0$)

Excess path (over $d=0$ case) = Γ

$$\Gamma = AC + CD - AB$$

$$AC = CD = \frac{d}{\cos \alpha}$$

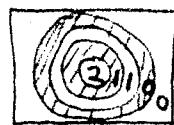
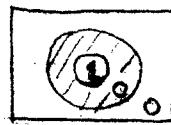
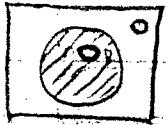
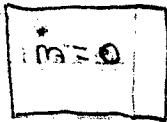
$$AB = 2AE \sin \alpha \quad AE = AC \sin \alpha = \cancel{d} \sin \alpha \\ AB = 2d \tan \alpha \sin \alpha = d \tan \alpha$$

$$\begin{aligned}\Gamma &= \frac{2d}{\cos \alpha} - 2d \tan \alpha \sin \alpha \\ &= \frac{2d}{\cos \alpha} (1 - \sin^2 \alpha) = \frac{2d \cos \alpha}{\cos^2 \alpha}\end{aligned}$$

(Compare to soap film $\Gamma = 2nd \cos \beta$)

$$\therefore \Gamma = 2d \cos \alpha = m\lambda \text{ for max} \\ \Gamma = 2d \cos \alpha = (m + \frac{1}{2})\lambda \text{ for min}$$

Note: no extra π phase shift here. π at each mirror



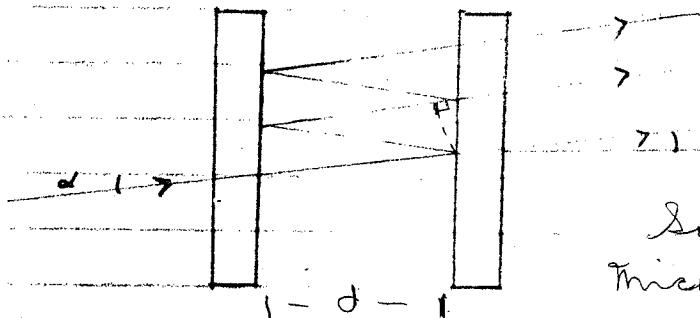
$$d=0$$

$$\Gamma = 0 \quad \Gamma = \frac{\lambda}{2} \cos \alpha \quad \Gamma = \lambda \cos \alpha \quad \Gamma = 2\lambda \cos \alpha$$

$$d=\lambda$$

$$d=\lambda$$

2) Fabry - Perot Interference
Interference Filter (Narrow band)



Same as
Michelson

$$2d \cos \alpha = m\lambda \quad \text{for max}$$

$$2d \cos \alpha = (m + \frac{1}{2})\lambda \quad \text{for min}$$

Note π phase changes at both
air-glass reflection surfaces

Applications -

Laser interferometry

Bug on water

Optical filter

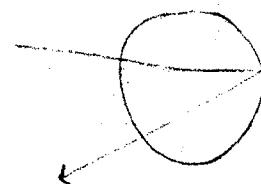
Some more thoughts

1) Why does a very thin (<<1) film appear black

Answer: Because on front surface there is a π phase shift and not in back. If very thin then reflected wave is very nearly cancelled.

2) Why is central Newton ring dark if some dust is between glass and light if optical grease is applied

3) Rainbow as refraction



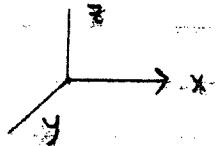
index of refraction of water varies with

Reflection - Optics and Q.M.

$$\nabla^2 \bar{E} - \mu \epsilon \frac{\partial^2 \bar{E}}{\partial t^2} = 0 \quad n = \sqrt{\mu \epsilon}$$

$$\mu = \kappa m_0, \quad \epsilon = \kappa_e \epsilon_0, \quad n = \sqrt{\kappa \kappa_e \epsilon_0} = \sqrt{\kappa} \quad n = \sqrt{\kappa}$$

General solution: $\bar{E} = \bar{E}_0 e^{i(\omega t + kx)}$



x - dir of prop

Assume wave is z axis polarized

$$\text{Let } \psi = E_z$$

$$\nabla^2 \psi - \mu \epsilon \frac{\partial^2 \psi}{\partial t^2} = 0$$

$$\psi = \psi_0 e^{i(\omega t + kx)} \quad k^2 - \omega^2 \frac{n^2}{c^2} = 0$$

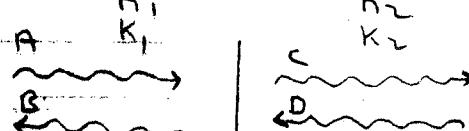
For $\nabla^2 \psi, \frac{\partial^2 \psi}{\partial t^2}$ to be finite $\psi, \nabla \psi$ must be continuous

Also $\oint \bar{E} \cdot d\vec{l} = -\frac{\partial \Phi_0}{\partial t} \rightarrow 0$ as path width shrinks $E_{||}$ continuous (E_z continuous)

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{n^2}{c^2} \frac{\partial^2 \psi}{\partial t^2}$$

$$\psi = \psi_0 e^{i(\omega t + kx)}$$

II) Plane slab



$$\psi = e^{i\omega t} (A e^{ikx} - B e^{-ikx})$$

$$\psi = e^{i\omega t} (C e^{-ikx} + D e^{ikx})$$

for $n_1 < n_2$ II phase shift for B
 $D = 0$ in our case

$$\text{Continuity of } \psi \rightarrow A - B = C$$

$$\text{Continuity of } \frac{\partial \psi}{\partial x} \rightarrow k_1(A + B) = k_2 C$$

at $x = 0$

at $x = c$

$$B + C = A$$

$$-k_1 B + k_2 C = k_1 A$$

$$B = \frac{\begin{vmatrix} A & 1 \\ -k_1 & k_2 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ -k_1 & k_2 \end{vmatrix}} = \frac{A(k_2 - k_1)}{k_2 + k_1}$$

$$C = \frac{\begin{vmatrix} 1 & A \\ -k_1 & -k_1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ -k_1 & k_2 \end{vmatrix}} = \frac{2Ak_1}{k_2 + k_1}$$

$$\text{recall } K^2 - \omega^2 \frac{n^2}{c^2} = 0 \quad K = \omega \frac{n}{c}$$
$$k_1 = \omega \frac{n}{c} \quad k_2 = \omega \frac{n}{c}$$

$$\therefore B = \frac{n_2 - n_1}{n_2 + n_1} A$$

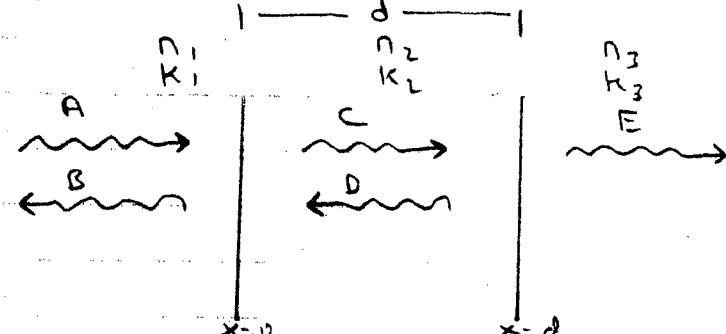
$$C = \frac{2n_1}{n_2 + n_1} A$$

flex $\propto E^2$

fractional power reflected = $\frac{B^2}{A^2}$

$$= \left(\frac{n_2 - n_1}{n_2 + n_1} \right)^2$$

2) Finite slab



Assume shift for B

$$n_1 < n_2 > n_3$$

IT phase

$$\psi = \begin{cases} A e^{-ik_1 x} - B e^{ik_1 x} & x < 0 \\ C e^{-ik_2 x} + D e^{ik_2 x} & 0 < x < d \\ E e^{-ik_3 x} & x > d \end{cases}$$

$$A - B = C + D$$

$$-k_1 A - K_1 B = -K_2 C + k_2 D$$

$$C e^{-ik_2 d} + D e^{ik_2 d} = E e^{-ik_3 d}$$

$$-k_2 C e^{-ik_2 d} + k_2 D e^{ik_2 d} = -K_3 E e^{-ik_3 d}$$

$$B = C + D = A$$

$$-k_1 A + k_2 C - k_2 D = k_1 A$$

$$e^{-ik_2 d} C + e^{ik_2 d} D - e^{-ik_3 d} E = 0$$

$$-k_2 C e^{-ik_2 d} + k_2 D e^{ik_2 d} + K_3 E e^{-ik_3 d} = 0$$

$$B =$$

$$\begin{vmatrix} A & 1 & 1 & 0 \\ K_1 A & k_2 & -K_2 & 0 \\ 0 & e^{-ik_2 d} & e^{ik_2 d} & -e^{-ik_3 d} \\ 0 & -k_2 e^{-ik_2 d} & k_2 e^{ik_2 d} & K_3 e^{-ik_3 d} \end{vmatrix}$$

$$\begin{vmatrix} 1 & 1 & 1 & 0 \\ -K_1 & k_2 & -K_2 & 0 \\ 0 & e^{-ik_2 d} & e^{ik_2 d} & -e^{-ik_3 d} \\ 0 & -k_2 e^{-ik_2 d} & k_2 e^{ik_2 d} & K_3 e^{-ik_3 d} \end{vmatrix}$$

$$\text{top} = A (k_2 e^{i(h_2 - h_3)d} (k_3 + k_2) + k_2 (k_3 - h_2) e^{i(h_2 + h_3)} \\ - K_A (2K_2 - e^{-i(h_2 + h_3)d} (k_3 + k_2)))$$

$$\text{bottom} = (K_2 e^{i(h_2 - h_3)d} (K_3 + k_2) + h_2 (k_3 - h_2) e^{-i(h_2 + h_3)d}) \\ + K_2 (2k_2 - e^{-i(h_2 + h_3)d} (h_2 + K_2))$$

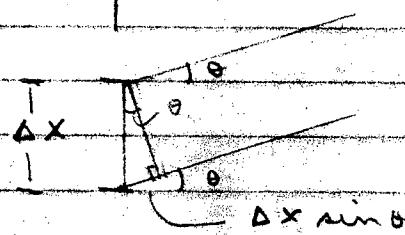
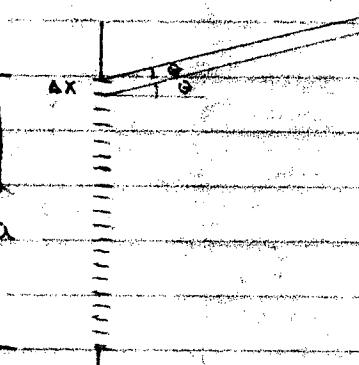
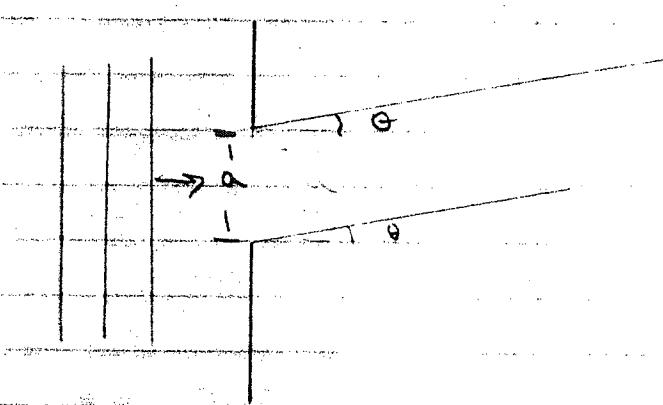
$$\text{top} = A \left[-2K_2 K_2 + K_2 (K_3 + K_2) e^{i(K_2 - K_3)d} \right. \\ \left. + (K_2 (K_3 - K_2) + K_2 (K_3 + K_2)) e^{-i(K_2 + K_3)d} \right]$$

$$\text{bottom} = [2K_2 K_2 + h_2 (h_3 + h_2) e^{i(K_2 - K_3)d} \\ + (h_2 (K_3 - h_2) - h_2 (K_3 + K_2)) e^{-i(K_2 + h_3)d}]$$

for case where $n_1 = n_3 = 1$ $n_2 = n$

$$\text{top} \propto A \left[-2n + n(1+n) e^{i(h_2 - h_3)d} + (1+2n-n^2) e^{-i(h_2 + h_3)d} \right]$$

Single Slit Diffraction | another use of interference



$$\Delta\phi = \frac{\Delta x \sin \theta}{\lambda} \quad \phi(x) = \frac{x \sin \theta}{\lambda}$$

$$\Delta E = E_0 \sin(\omega t + \phi) \frac{\Delta x}{a}$$

$$E = \sum \Delta E \rightarrow \int dE = E_0 \int_0^a \sin\left(\omega x + \frac{x \sin \theta}{\lambda}\right) \frac{dx}{a}$$

$$= -E_0 \left[\frac{2\pi a \sin \theta}{\lambda} \cos\left(\omega x + \frac{x \sin \theta}{\lambda}\right) \right] \Big|_0^a$$

$$\cos A = \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\therefore E = \frac{-E_0}{2\pi a \sin \theta} \left(\cos(\omega t + \frac{\pi a \sin \theta}{\lambda}) - \cos \right)$$

$$= \frac{E_0}{\pi a \sin \theta} \left[\sin(\omega t + \frac{\pi a \sin \theta}{\lambda}) \sin \frac{\pi a \sin \theta}{\lambda} \right]$$

$$= E_0 \sin(\omega t + \alpha) \frac{\sin \alpha}{\alpha}$$

$$\alpha = \frac{\pi a}{\lambda} \sin \theta$$

$$\rightarrow I = I_0 \left(\frac{\sin \alpha}{\alpha} \right)^2 \quad \frac{\sin \alpha}{\alpha} \rightarrow 1 \quad \alpha \rightarrow$$

Central maximum for $\alpha = 0 \rightarrow \theta = 0$

Secondary min $\alpha = m\pi$ $a \sin \theta = m$
 Secondary max $\alpha = (m + \frac{1}{2})\pi$ $a \sin \theta = (m + \frac{1}{2})$

Maxima $\alpha = (m + \frac{1}{2})\pi$

$$I_m = I_0 \cdot \frac{1}{(m + \frac{1}{2})^2 \pi^2}$$

$$\frac{I_m}{I_0} \quad m$$

| | |
|--------|---|
| 0.045 | 1 |
| 0.016 | 2 |
| 0.0083 | 3 |

Width of peak.

first minimum $m = 1$

$$a \sin \theta_p = m\lambda = \lambda$$

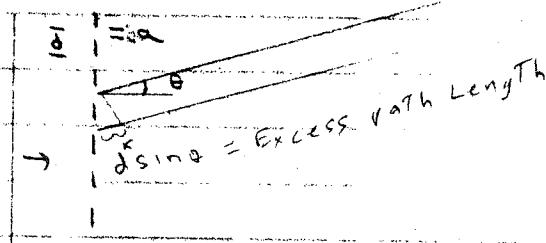
$$a \sin \theta_p \approx a \theta_p \approx \lambda$$

$$\theta_p \approx \lambda/a$$

General property of diffraction:

Note as "a" becomes smaller
 θ gets larger. This sets a
lower limit on the size of a
spot thru a hole.

Diffraction Grating



$$1) d \sin \theta = (m + \frac{1}{2})\lambda \text{ for min}$$

$$d \sin \theta = m\lambda \text{ for max}$$

$$m = 0, \pm 1, \dots$$

m = order number

d = distance between slits
 a = slit size

1) Recall for diffraction thru a single slit first min occurs for $a \sin \theta = m\lambda$ $m=1$

$$\theta_p \sim \frac{\lambda}{a}$$

2) First min for grating occurs for $d \sin \theta = (m + \frac{1}{2})\lambda$ $m=0$

$$\theta_p \sim \frac{\lambda}{2d} \quad \theta \text{ small (rad)}$$

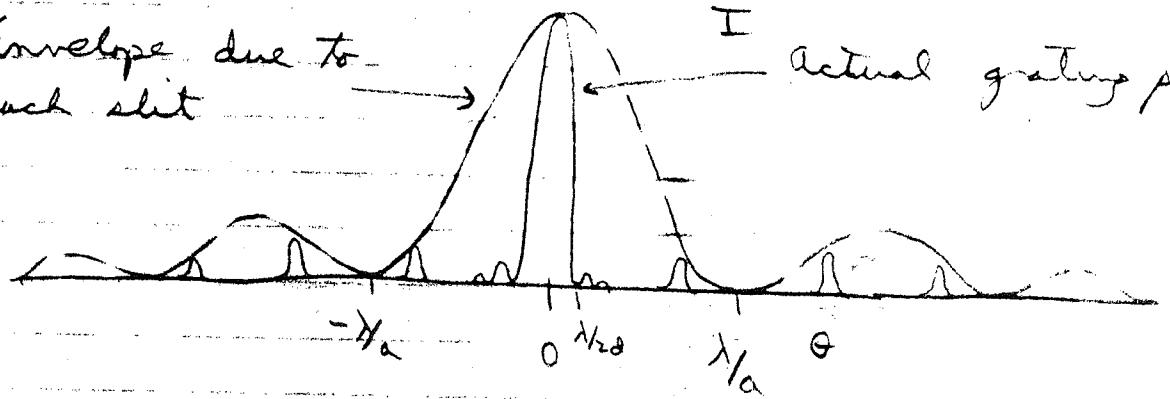
But in general $a \ll d$
 so $\theta_p \gg \theta_d$

So diffraction pattern from grating is modulated by diffraction pattern from each slit.

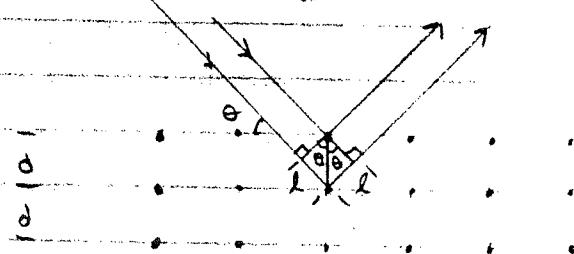
Envelope due to
each slit

I

actual grating pattern



Bragg scattering - X rays



Interplanar spacing d of atoms in a cyl.

$$l = d \sin \theta$$

$$\Gamma = \text{Phase difference} = 2l = 2d \sin \theta$$

Constructive interference for $\Gamma = m\lambda$

$$2d \sin \theta = m\lambda \quad m = 0, 1, 2, 3, \dots$$

1) knowing $\lambda \rightarrow d$

2) knowing $d \rightarrow \lambda$

3) Broad X-ray distribution \rightarrow monochromatic reflected (diffracted) beam