

Physics 128 – Senior Lab

<http://www.physics.ucsb.edu/~phys128/>

<http://www.deepspace.ucsb.edu/classes/>

Class Gmail account:

ucsb.physics128a@gmail.com Pass: @physics128a

ucsb.physics128b@gmail.com Pass: @physics128b

Please add your contact info (email, tel) to Contacts

Staff - Bob Pizzi
(Broida 3310)

TA's:

Dorsett, Alex	adorsett20@gmail.com
Frey, Johnathon	jfrey@physics.ucsb.edu
Langman, Eric	elangman@physics.ucsb.edu

PSR – Broida 1019 – Computer Lab

Textbooks – Bookstore – Optional
Data Reduction and Error Analysis - Bevington
Phys 128 Lab Manual

First week

Broida 3223 or 3340 depending on AV status
1-3 PM
otherwise labs are in 3223

Error analysis, lab overview

What is Senior Lab?

Is it a pain?

Is it fair?

Is it fun?

Will you regret it?

**Senior lab second quarter sequence
consists of four labs done in a sequence of
2 weeks of lab and analysis and writeup.
Writeup due following week. Labs start
next week.**

**Please write down all labs you did in
128A.**

Physics 128A MW 1-5:50

Physics 128B TTh 1-5:50

Lab start dates: 10/3, 10/17, 10/31, 1/14

Machine Shop - In addition you should have done a project in the machine shop.

Prereq – Junior or senior standing.
Physics majors only. Phys 115 (QM) finished or currently enrolled. NMR, Mossbauer require some QM

You will have to keep a lab notebook.

You will have two lab books – rotate using one and having the other graded. Keeping a good lab book (organized, methodical) is critical.

ALL materials **MUST** be stored on the lab server under your name (sub dir) and with each lab being its own sub dir **UNDER** your name sub dir (ie \senior lab Fall 2016a (or b)\john doe\Project 1

In addition to the notebook you will turn in a paper (that MUST also be on the server in editable form (MS Office or Open Office) AND final PDF - NOT just PDF) for each lab.

The paper:

Theory – physics behind the experiment.

Experimental Technique

Data summary

Analysis including error analysis

Conclusions – what worked what failed

What you would change about the lab

Verbal presentations required

At start of 2 week sequence writeup (on transparency) a very short summary of the theory of the lab, equipment needed and how you plan to proceed.

There are 14 labs to choose from. You cannot repeat a lab you already did.

Labs

HD Isotope shift

Interferometer

Laser Properties

Mossbauer

Pulsed Nuclear Magnetic Resonance

Muon Physics

Muon Tomography

Cloud Chamber

Electron Properties

Gamma Ray Spectroscopy

Gravitational Constant - Cavendish experiment

Microwave Optics

3-D AutoCAD

Diode Laser Spectroscopy

Magnetic Fields

PHYS 128 AL SENIOR LAB 2016-2017			9/19/2016		
			128AL	128AL/BL	128AL
					STUDENTS
LAB	ROOM		F '16	W '17	S '17
			2/LAB	2/LAB	2/LAB
	SECT./QTR.		2	1	2
CAVENDISH BALANCE, BIG "G"	3223		2	2	2
CLOUD CHAMBER	3215		2	2	2
GAMMA RAY SPECTROSCOPY	3215		2	2	2
H-D SHIFT	3207		2	2	2
INTERFEROMETER	3207		2	2	2
LASER PROPERTIES	3207		2	2	2
HOLOGRAPHY	RETIRED				
MAGNETIC FIELDS	3223		2	2	2
MASS OF THE ELECTRON (e/m, e)	3223		2	2	2
MICROWAVE OPTICS	3223		2	2	2
MOSSBAUER EFFECT	3207		2	2	2
MUON PHYSICS	3215		2	2	2
MUON TOMOGRAPHY (NEW)	3223		2	2	2
PNMR	3215		2	2	2
3-D AUTO CAD	3217		2	2	2
VACUUM TECHNIQUES	RETIRED				
DIODE LASER SPECTROSCOPY (N)	3209		2	2	2
TOTAL LABS			30	30	30
STUDENTS/ SECTION			30	30	30
QUARTER TOTAL			60	30	60



Electron Properties

[Charge to Mass Ratio \[PDF\]](#)
[Oil Drop \[PDF\]](#)



Cavendish Torsion Balance (Big G)

[Lab Manual \[PDF\]](#)



Cloud Chamber

[Lab Manual \[PDF\]](#)



Gamma Ray Spectroscopy

[Gamma Ray Intro \[PDF\]](#)
[UCS 30 Manual \[PDF\]](#)
[Gamma Ray Lab Manual \[PDF\]](#)



H-D Isotope Shift

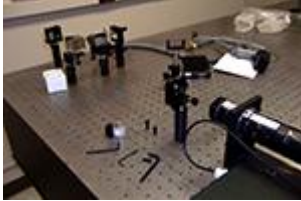
[Lab Manual \[PDF\]](#)



Holography

[Lab Manual \[PDF\]](#)
[Supplement \[PDF\]](#)
[Revised Drawing \[PDF\]](#)





Interferometry

[Lab Guide \[PDF\]](#)
[Supplement \[PDF\]](#)



Laser Properties

[Lab Manual \[PDF\]](#)
[Addendum \[PDF\]](#)



Microwave Optics

[Lab Manual \[PDF\]](#)



Mossbauer Effect

[Lab Manual \[PDF\]](#)



Muon Physics

[Lab Manual \[PDF\]](#)



Nuclear Magnetic Resonance

[University of Washington Paper \[PDF\]](#)

[Conceptual Tour \[PDF\]](#)

A good overview.



Noise

[Lab Manual \[PDF\]](#)
[Some Theory \[PDF\]](#)



Lab Partners – group of two per lab, perhaps three if odd number.

Course Materials

1. A lab notebook. This must be a *bound* (not looseleaf, not perforated) notebook, containing *quad-ruled paper* (*aka graph paper*). You will need TWO of them, since the TAs will be grading one lab while you need to be writing up the next one. Access to the lab writeups. The bookstore sells a bound copy or:

<http://www.physics.ucsb.edu/~phys128/>

You will also NEED to scan your lab notes for EACH lab and put it in your server area.

2. Access to Wolfram's *Mathematica* software. Installed on all lab computers, many UCSB computer-cluster computers, and in the PSR. If you want a copy on a personal machine, you can buy a (\$45) one-semester or (\$74) one-year temporary license, or a (\$140) regular copy, via <http://store.wolfram.com/catalog/>. There is no physics department or UCSB license with additional discounts.

3. Keep your data on the lab server NOT on a local machine drive. You can additionally use a USB thumb drive if you want to work on your data at home.

4. A sourcebook for statistics and data analysis.

Recommend [An Introduction to Error Analysis: The Study of Uncertainties in Physical Measurements](#), by John Taylor.

Schedule

Fall

WEEK	Dates	Activity
1	9/26	Lecture
2	10/3	Lab 1 start
3	10/10	Lab 1 continue
4	10/17	Lab 1 lab book due, paper due - Lab 2 start
5	10/24	Lab 2 continue
6	10/31	Lab 2 lab book due, paper due - Lab 3 start
7	11/7	Lab 3 continue
8	11/14	Lab 3 lab book due, paper due - Lab 4 start
9	11/21	Lab 4 continue
10	11/28	Lab 4 lab book due, paper due - complete any unfinished work

Labs

HD Isotope shift

Hydrogen has three isotopic states – the normal hydrogen nucleus is one proton and no neutrons, Two other nuclear isotopes are possible – Deuterium (D) has one proton and one neutron and Tritium (T) has one proton and two neutrons. D occurs on Earth at 154 ppm ($\sim 1/6500$) in water. D is stable (not radioactive) T is radio active 12.32 year half life \rightarrow decays to $\text{He-3} + e + \text{anti} - e - \text{neutrino}$
H, D, T have different nuclear masses \rightarrow reduced mass in atom (nucleus + e) is different \rightarrow electronic structure (spectrum) is different.

Purpose of Experiment is to measure this spectral difference - known as an isotope shift

Holography

[Greek](#), *όλος*-*hòlòs* whole + *γραφή*-*grafè* write

Invented in 1947 by [Hungarian](#) physicist [Dennis Gabor](#) (1900–1979). [Nobel Prize in physics](#) in 1971.

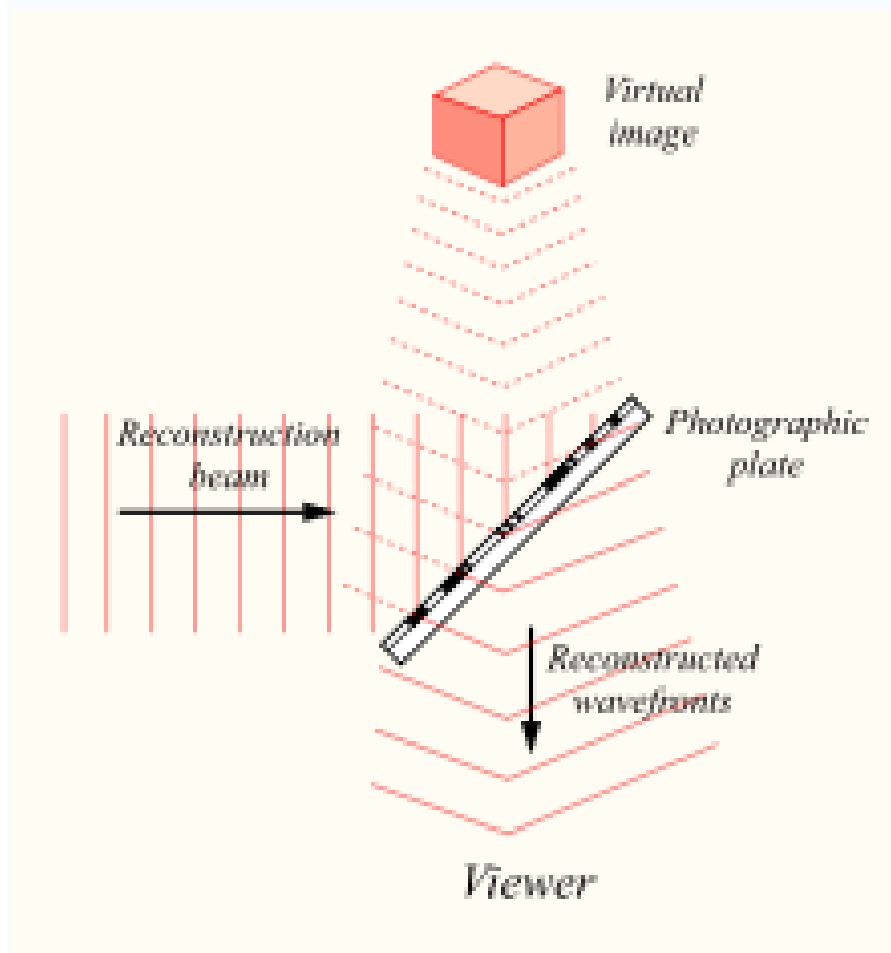
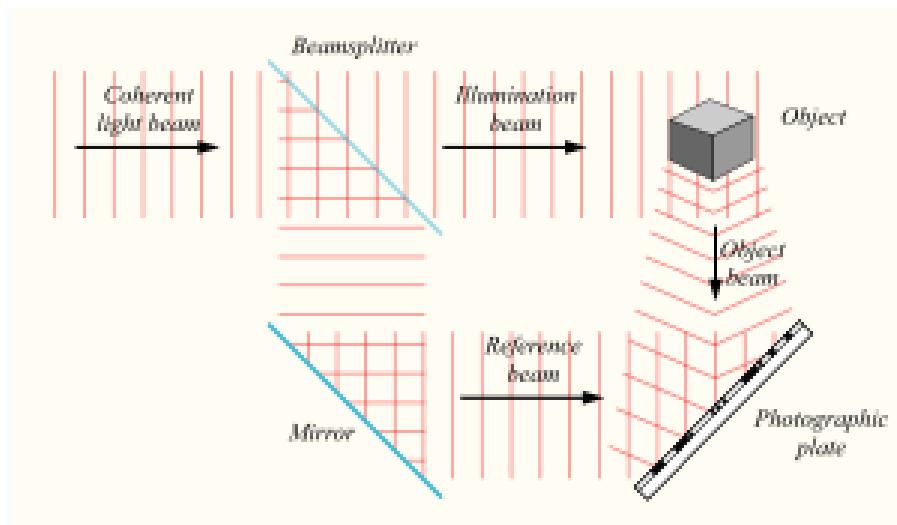
First 3D holograms were made by [Yuri Denisyuk](#) in the USSR in 1962 and slightly later by [Emmett Leith](#) and [Juris Upatnieks](#) at [University of Michigan](#) in 1962.

Note that Holography was first developed in 1947 BEFORE lasers were developed in 1960's.

Holography only became useful after lasers were developed though white light holography is possible.

Holography is different than normal imaging in that phase information is encoded in the imaging process rather than just intensity. Hence phase coherent illumination is needed. Holograms are like diffraction gratings. Two types of holograms are transmission (Leith and Upatnieks) and reflection (Denisyuk).

Recording and reconstruction are as follows.



Holograms have tremendous practical value. **Holographic**
For example the **Versatile Disc (HVD)** is an optical disc which can hold up to 4 TB – 160 times that of Blu Ray HD-DVD.

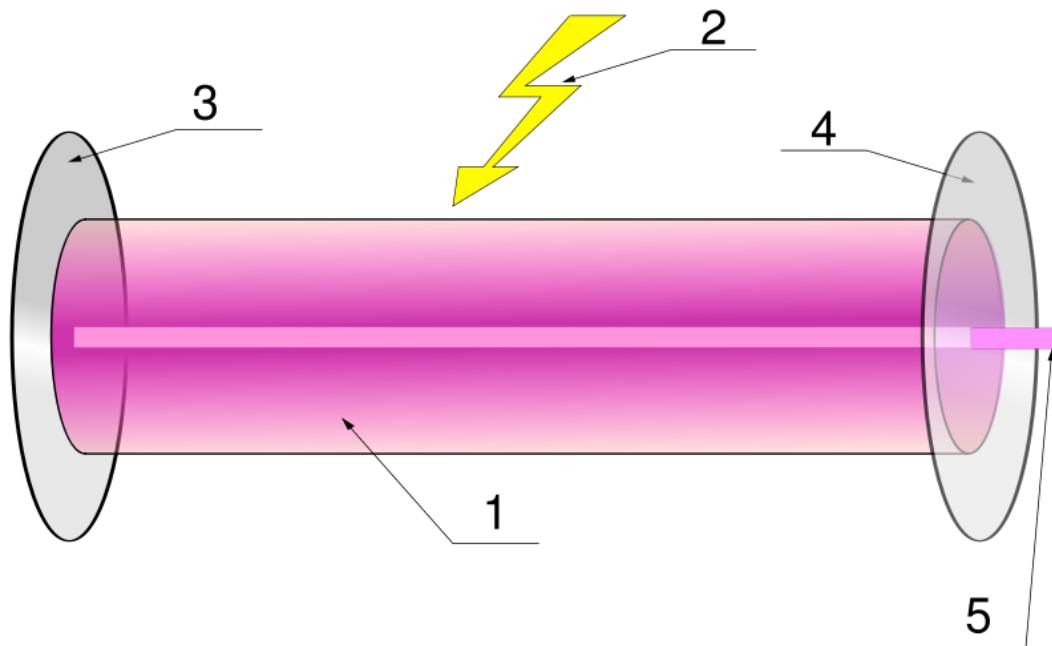
Laser properties

LASER = Light Amplification by Stimulated Emission of Radiation

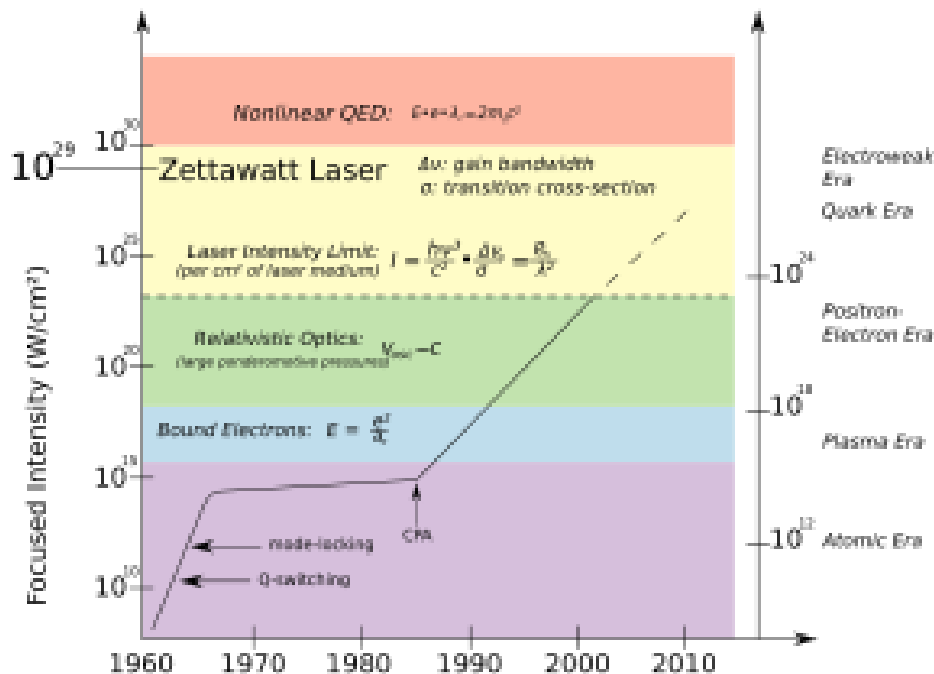
The first working laser was made in May 1960 by [Theodore Maiman](#) at [Hughes Research Laboratories](#).

LASER's were preceded by the development of MASER's (Microwave Amplification by Stimulated Emission of Radiation) - Townes et al – Nobel Prize 1964

LASER's use coherent amplification by inverted population states.



Very active field of research with enormous commercial value. LASER's are also used in military applications and "LASER blasters" may become reality in the near future. Inertial fusion, laser missile defense, ...



Mossbauer

The **Mössbauer effect** ([German](#): *Mößbauer* [*Meß-Bauer*]), discovered by [Rudolf Mößbauer](#) in [1957](#), the resonant and recoil-free emission and absorption of [gamma rays](#) by atoms bound in a solid.

[Nobel Prize in Physics](#) in [1961](#) together with [Robert Hofstadter](#)'s research of [electron scattering](#) in atomic nuclei.

The fundamental discovery was that some systems behave as though an individual radioactive nuclei behaves as though it is “locked” in the macroscopic matrix of the bulk material and not like an individual nucleus. Hence when it decays it does not recoil very

much. Sometimes called “**Recoilless emission and absorption**”.

Mossbauer gamma rays produced by nuclear transitions: from a high-energy state, to a stable low-energy state. The energy of the emitted gamma ray corresponds to the energy of the nuclear transition, **minus an amount of energy that is lost as recoil to the emitting atom.**

If the lost "recoil energy" is small compared with the energy [linewidth](#) of the nuclear transition, then the gamma ray energy still corresponds to the intrinsic energy of the nuclear transition, and the gamma ray can be absorbed by a second atom of the same type as the first. This emission and subsequent absorption is called **resonance**. Additional recoil energy is also lost during absorption, so in order for resonance to occur the recoil energy must actually be less than half the linewidth for the corresponding nuclear transition.

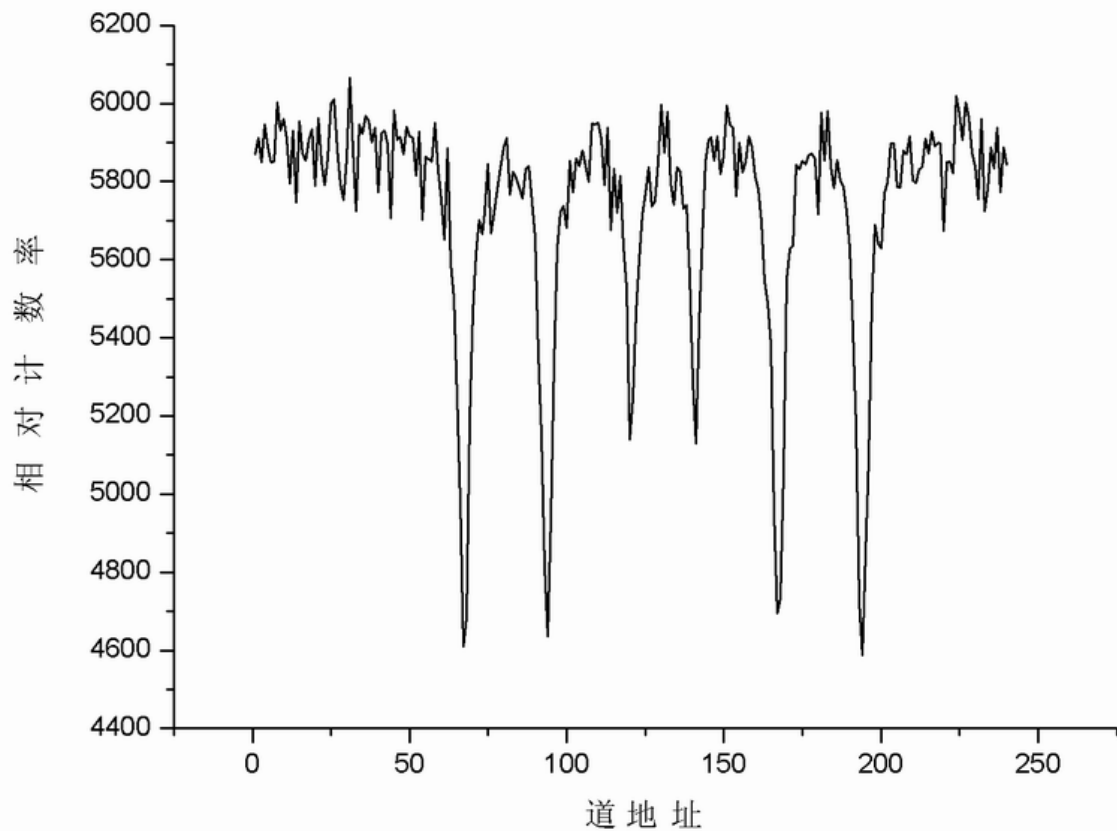
The amount of lost energy is described by the equation:

$$E_R = \frac{E_\gamma^2}{2Mc^2}$$

where E_R is the energy lost as recoil, E_γ is the energy of the gamma ray, M is the mass of the emitting or absorbing body. In the case of a gas the emitting and absorbing bodies are atoms, so the mass is small, resulting in a large recoil energy (note $1/M$ dependence), which prevents resonance. The same equation applies for recoil energy losses in x-rays, but the photon energy is much less, resulting in a lower energy loss, which is why gas-phase resonance could be observed with x-rays. Note E_γ^2 dependence.

In a solid, the nuclei are bound to the lattice and do not recoil in the same way as in a gas. The lattice as a whole recoils but the recoil energy is negligible because the M in the above equation is the mass of the whole lattice. However, the energy in a decay can be taken up (or supplied by) lattice vibrations. The energy of these vibrations is quantised in units known as *phonons*. The Mössbauer effect occurs because there is a finite probability of a decay occurring involving no phonons. Thus in a fraction of the nuclear events (the **recoil-free fraction**), the entire crystal acts as the recoiling body, and these events are essentially recoil-free. In these cases, since the recoil energy is negligible, the emitted gamma rays have the appropriate energy and resonance can occur.

Some (depending on the half-life of the decay), gamma rays have very narrow linewidths. Thus they are very sensitive to small changes in the energies of nuclear transitions. Gamma rays can be used as a probe to observe the effects of interactions between a nucleus and its electrons and those of its neighbors. This is the basis for [Mössbauer spectroscopy](#), which combines the Mossbauer effect with the [Doppler effect](#) to monitor such interactions.



Iron 57 Mossbauer spectrum – absorption spectrum 14.7 Kev line

Vertical axis is photon count, horizontal is velocity bin

Some Mossbauer application

Pound-Rebka experiment is an experiment to test [Einstein's](#) theory of [general relativity](#). It was proposed and tested by R. V. Pound and G. A. Rebka Jr. in [1959](#),^[1] It is a [gravitational redshift](#) experiment, measuring the [redshift](#) of gamma rays in a

gravitational field, or, equivalently, a test of the general relativity prediction that clocks should run at different rates at different places in a [gravitational field](#). Considered to be the experiment that ushered in an era of *precision* tests of general relativity.

When an atom transits from an excited state to a base state, it emits a photon with a specific frequency and energy. When the same atom in its base state encounters a photon with that same frequency and energy, it will absorb that photon and transit to the excited state. If the photon's frequency and energy is significantly different (greater than the linewidth) the atom cannot absorb it. When the photon travels through a gravitational field, its frequency and therefore its energy will change due to the gravitational redshift. Thus as a result the receiving atom can no longer absorb it if the gravitation shift shifts the energy outside the linewidth.

If you move the emitting atom with just the right speed relative to the receiving atom the resulting [doppler shift](#) will cancel out the gravitational red shift and the receiving atom will be able to absorb the photon (resonance). The relative speed of the atoms is therefore a measure of the gravitational redshift.

Pound-Rebka used an elevator shaft at Harvard. The height was 22.5 meters. The fractional change in energy is given by $\delta E/E = gh/c^2 = 2.5 \times 10^{-15}$. This is very small and is where the Mossbauer effect becomes appropriate. The 14 keV [gamma rays](#) emitted by [iron-57](#) when it transitions to its base state proved to be sufficient for this experiment.

A solid sample containing gamma rays emitting [iron](#) (^{57}Fe) was placed in the center of a [loudspeaker](#) cone which was placed near the roof of the building. Another sample containing ^{57}Fe was placed in the basement. **The gamma rays traveled through a [Mylar](#) bag filled with helium to minimize scattering of the gamma rays.** A [scintillation counter](#) was placed below the receiving ^{57}Fe sample to detect the gamma rays that were not absorbed by the receiving sample. By vibrating the speaker cone the gamma ray source moved with varying speed, thus creating varying doppler shifts. When the doppler shift canceled out the gravitational redshift, the receiving sample absorbed gamma rays and the number of gamma rays detected by the scintillation counter dropped accordingly. The variation in absorption was correlated with the speed of the speaker and thus the doppler shift. To compensate for possible [systematic errors](#), Pound and Rebka varied the speaker frequency between 10 Hz and 50 Hz,

interchanged the source and absorber-detector, and used different speakers (ferroelectric and moving coil magnetic [transducer](#)).

The result confirmed that the predictions of general relativity were borne out at the 10% level. This was later improved to better than the 1% level by Pound and Snider.

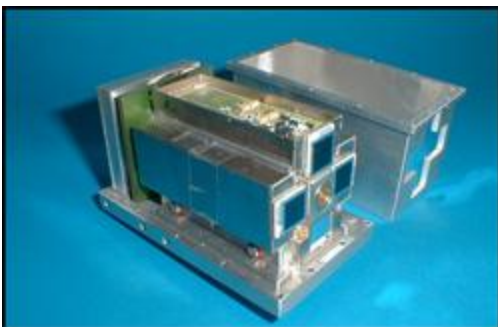
Later another test involving a [hydrogen maser](#) increased the accuracy of the measurement to about 10^{-4} .



Harvard Jefferson Lab – Pound-Rebka used attic to basement shaft.

Mars Rover – Mossbauer Spectrometer

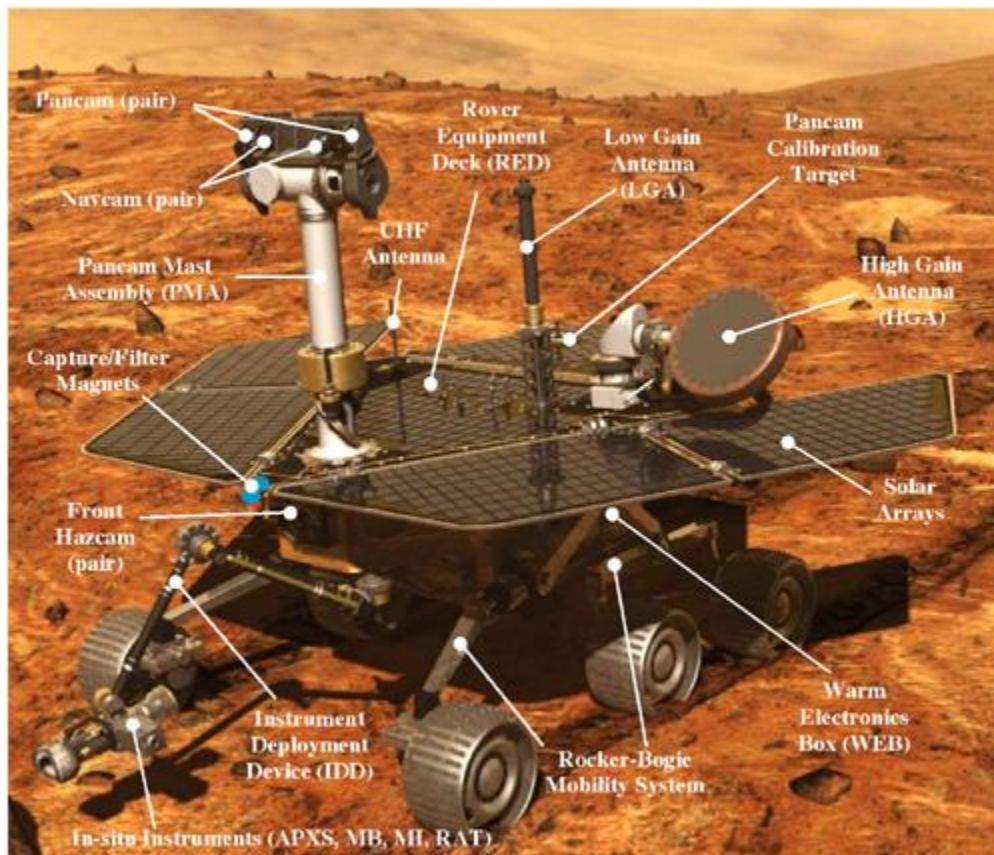
Mössbauer Spectrometer (MB)



Many of the minerals that formed rocks on Mars contain iron, and the soil is iron-rich. The Mössbauer Spectrometer is an instrument that was specially designed to study iron-bearing minerals. Because this science instrument is so specialized, it can determine the composition and abundance of these minerals to a high level of

accuracy. This ability can also help us understand the magnetic properties of surface materials.

The Mössbauer Spectrometer sensor head is small enough to fit in the palm of your hand. It is one of four instruments mounted on the turret at the end of the [rover arm](#). Its electronics are housed inside the [body of the rover](#) (in the Warm Electronics Box, or WEB). Measurements are taken by placing the instrument's sensor head directly against a rock or soil sample. One Mössbauer measurement takes about 12 hours.



Nuclear Magnetic Resonance (NMR)

This is a very important effect with wide use in medicine and industry.

Nuclear magnetic resonance (NMR) is a [quantum mechanical magnetic](#) property of an [atom](#)'s [nucleus](#). NMR also commonly refers to a family of scientific methods that exploit nuclear magnetic resonance to study [molecules](#).

All nuclei that contain odd numbers of [protons](#) or [neutrons](#) have an intrinsic [magnetic moment](#) and [angular momentum](#). The most commonly measured nuclei are hydrogen and [carbon-13](#), although nuclei from isotopes of many other elements (e.g. ^{15}N , ^{14}N , ^{19}F , ^{31}P , ^{17}O , ^{29}Si , ^{10}B , ^{11}B , ^{23}Na , ^{35}Cl , ^{195}Pt) can also be observed.

NMR resonant frequencies for a particular substance are directly proportional to the strength of the applied magnetic field, in accordance with the equation for the [Larmor precession](#) frequency.

$$\vec{\Gamma} = \vec{\mu} \times \vec{B} = \gamma \vec{J} \times \vec{B}$$

where γ is the gyromagnetic ratio which is the proportionality constant between the magnetic moment and the angular momentum. The Larmor frequency is:

$$\omega = \gamma B$$

Classical Charged Body Gyromagnetic Ratio

Consider a [charged](#) body rotating about an axis of symmetry. It has both a magnetic dipole moment and an angular momentum on account of its rotation. As long as its charge and mass are distributed identically (e.g., both distributed uniformly), its gyromagnetic ratio is

$$\gamma = \frac{q}{2m}$$

where q is its charge and m is its mass. The derivation of this relation is as follows:

Consider an infinitesimally narrow circular ring within the body (the general case follows from integration). Suppose the ring has radius r , area $A = \pi r^2$, mass m , charge q , and angular momentum

$L = mvr$. Then the magnitude of the magnetic dipole moment is

$$\mu = IA = \frac{qv}{2\pi r} * \pi r^2 = \frac{q}{2m} * mvr = \frac{q}{2m} L$$

Thus:

$$\gamma = \frac{q}{2m}$$

Gyromagnetic ratio for an isolated electron

An isolated electron has an angular momentum and a magnetic moment resulting from its [spin](#). This is not a classical effect as the electron is not a classical structure. There is no reason to expect the above classical relation to hold. In fact it does not, giving the wrong result by a dimensionless factor called the electron [g-factor](#), denoted g_e or just g when there is no risk of confusion with gravity:

$$\gamma_e = \frac{-e}{2m_e} g_e = -g_e \mu_B / \hbar,$$

where μ_B is the [Bohr magneton](#)

$$\mu_B = \frac{e\hbar}{2m_e}.$$

The electron g-factor g_e is a bit more than two, and has been measured to twelve decimal places:

$$g_e = 2.0023193043617(15)$$

(note the 15 is the error in the last two digits – error analysis)

The electron gyromagnetic ratio is given by NIST (National Institute of Science and Technology – Boulder – formerly NBS (National Bureau of Standards) as

$$\gamma_e = 1.760\,859\,770(44) \times 10^{11} \text{ s}^{-1} T^{-1}.$$

Gyromagnetic ratio for a nucleus

Protons, neutrons, and many nuclei carry [nuclear spin](#), which gives rise to a gyromagnetic ratio as above. The ratio is conventionally written in terms of the proton mass and charge, even for neutrons and for other nuclei, for the sake of simplicity and consistency. The formula is:

$$\gamma = \frac{e}{2m_p} g = g\mu_N/\hbar,$$

where μ_N is the [nuclear magneton](#), and g is the [g-factor](#) of the nucleon or nucleus in question.

The gyromagnetic ratio of a nucleus is particularly important because of the role it plays in (NMR) and [Magnetic Resonance Imaging](#) (MRI). These procedures rely on the fact that nuclear spins [precess](#) in a magnetic field at the [Larmor frequency](#), which is the product of the gyromagnetic ratio with the magnetic field strength.

Approximate values for some common nuclei are given in the Table below.

Nucleus	$\gamma / 2\pi$ (MHz/T)
^1H	42.576
^3He	-32.434
^7Li	16.546
^{13}C	10.705
^{14}N	3.0766

^{15}N	-4.3156
^{17}O	-5.7716
^{23}Na	11.262
^{31}P	17.235
^{129}Xe	-11.777

NMR studies magnetic nuclei by aligning them with an applied constant [magnetic field](#) and perturbing this alignment using an alternating electric field, those fields being [orthogonal](#). The resulting response to the perturbing magnetic field is the phenomenon that is exploited in [NMR spectroscopy](#) and [magnetic resonance imaging](#), which use very powerful applied magnetic fields in order to achieve high resolution spectra, details of which are described by the [chemical shift](#) and the [Zeeman effect](#).

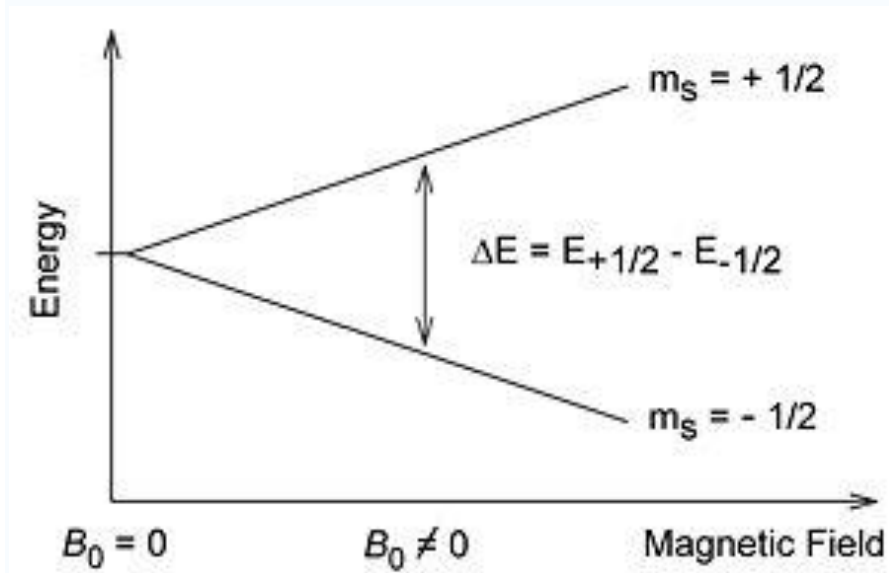


Northwest National Lab – 800 MHz NMR – note superconducting magnet dewar

Spin behavior in a magnetic field

Consider nuclei which have a spin of one-half, like ^1H , ^{13}C or ^{19}F . The nucleus has two possible spin states: $m = 1/2$ or $m = -1/2$ (also referred to as up and down or α and β , respectively). The energies of these states are degenerate—that is to say that they are the

same. Hence the *populations* of the two states (i.e. number of atoms in the two states) will be exactly equal at [thermal equilibrium](#).



If a nucleus is placed in a magnetic field, however, the interaction between the nuclear magnetic moment and the external magnetic field mean the two states no longer have the same energy. The [energy](#) of a magnetic moment μ when in a magnetic field B_0 (the zero subscript is used to distinguish this [magnetic field](#) from any other applied field) is given by the negative scalar product of the vectors:

$$E = -\mathbf{B}_0 \cdot \mu = -\mu_z B_0$$

where the magnetic field has been oriented along the z axis.

Hence

$$E = -m\hbar\gamma B_0$$

As a result the different nuclear spin states have different energies in a non-zero magnetic field. In [hand-waving](#) terms, we can talk about the two spin states of a spin $\frac{1}{2}$ as being *aligned* either with or against the magnetic field. If γ is positive (true for most isotopes) then $m = \frac{1}{2}$ is the lower energy state.

The energy difference between the two states is

$$\Delta E = \hbar\gamma B_0$$

and this difference results in a small population bias toward the lower energy state.

Resonance

Resonant absorption will occur when [electromagnetic radiation](#) of the correct frequency to match this energy difference is applied. The energy of a [photon](#) is $E = h\nu$, where ν is its frequency. Hence absorption will occur when

$$\nu = \frac{\Delta E}{h} = \frac{\gamma B_0}{2\pi}$$

These frequencies typically correspond to the [radio frequency](#) range of the [electromagnetic spectrum](#).

It is this resonant absorption that is detected in NMR.

Nuclear shielding

It might appear from the above that all nuclei of the same nuclide (and hence the same γ) would resonate at the same frequency. This is not the case. The most important perturbation of the NMR frequency for applications of NMR is the 'shielding' effect of the surrounding electrons. In general, this electronic shielding reduces the magnetic field *at the nucleus* (which is what determines the NMR frequency). As a result the energy gap is reduced, and the frequency required to achieve resonance is also reduced. This shift of the NMR frequency due to the chemical environment is called the [chemical shift](#), and it explains why NMR is a direct probe of chemical structure.

Unless the local [symmetry](#) is particularly high, the shielding effect depends on the orientation of the molecule with respect to the external field. In [solid-state NMR](#), [magic angle spinning](#) is required to average out this orientation dependence. This is unnecessary in conventional NMR of molecules in solution since rapid molecular tumbling averages out the anisotropic component of the chemical shift.

Relaxation

The process called population relaxation refers to nuclei that return to the thermodynamic state in the magnet. This process is also called T_1 relaxation, where T_1 refers to the mean time for an individual nucleus to return to its equilibrium state. Once the population is relaxed, it can be probed again, since it is in the initial state.

The [precessing](#) nuclei can also fall out of alignment with each other (returning the net magnetization vector to a nonprecessing field) and stop producing a signal. This is called T_2 relaxation. It is possible to be in this state and not have the population difference required to give a net magnetization vector at its thermodynamic state. Because of this, T_1 is always larger (slower) than T_2 . This happens because some of the spins were flipped by the pulse and will remain so until they have undergone population relaxation. In practice, the T_2 time is the life time of the observed NMR signal, the [free induction decay](#). In the NMR spectrum, meaning the [Fourier transform](#) of the [free induction decay](#), the T_2 time defines the width of the NMR signal. Thus, a nucleus having a large T_2 time gives rise to a sharp signal, whereas nuclei with shorter T_2 times give rise to more broad signals. The

length of T_1 and T_2 is closely related to molecular motion.

Noise Properties in thermal equilibrium

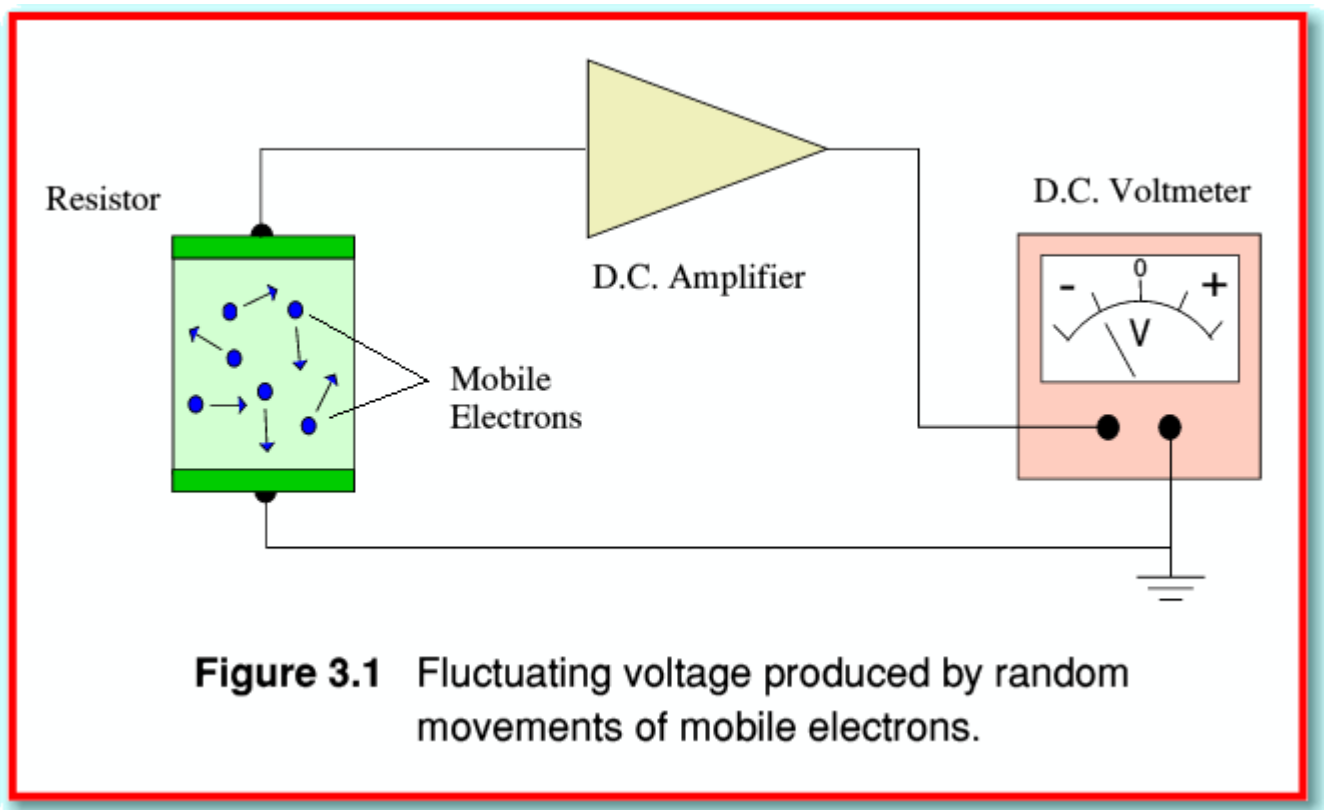
Johnson noise

In 1927 J. B. Johnson observed random fluctuations in the voltages across electrical resistors. A year later H. Nyquist published a theoretical analysis of this noise which is thermal in origin. Hence this type of noise is variously called ***Johnson noise***, ***Nyquist noise***, or ***Thermal noise***.

Consider a resistor as consisting of conductive material with two electrical contacts. In order to conduct electricity the material must contain some charges which are free to move. Treat it as ‘box’ of material which contains mobile electrons (charges) which move around, interacting with each other and with the atoms of the material. At any non-zero temperature we can think of the moving charges as a sort of ***Electron Gas*** trapped inside the resistor box.

The electrons move about in a randomised way — similar to Brownian motion — bouncing and scattering off one another and the atoms. At any particular instant there may be more electrons near one end of the box than the other. This means there will be a difference in electric potential between the ends of the box (i.e. the non-uniform charge distribution produces a voltage across the resistor).

As the distribution fluctuates from instant to instant the resulting voltage will also vary unpredictably.



In the figure a resistor is connected via an amplifier to a centre-zero d.c. voltmeter. The meter

readings move randomly in response to the thermal movements of the charges within the resistor.

We can't predict what the precise noise voltage will be at any future moment (**Random Phase**). We can however make some **statistical** predictions after observing the fluctuations over a period of time.

We can plot a histogram of the results. Choose a voltage 'bin width', dV , and divide up the range of possible voltages into small 'bins' of this size. We then count up how often the measured voltage was in each bin, divide those counts by the total number of measurements, and plot a histogram of the form shown below.

We can use this plot to indicate the likelihood or *probability*

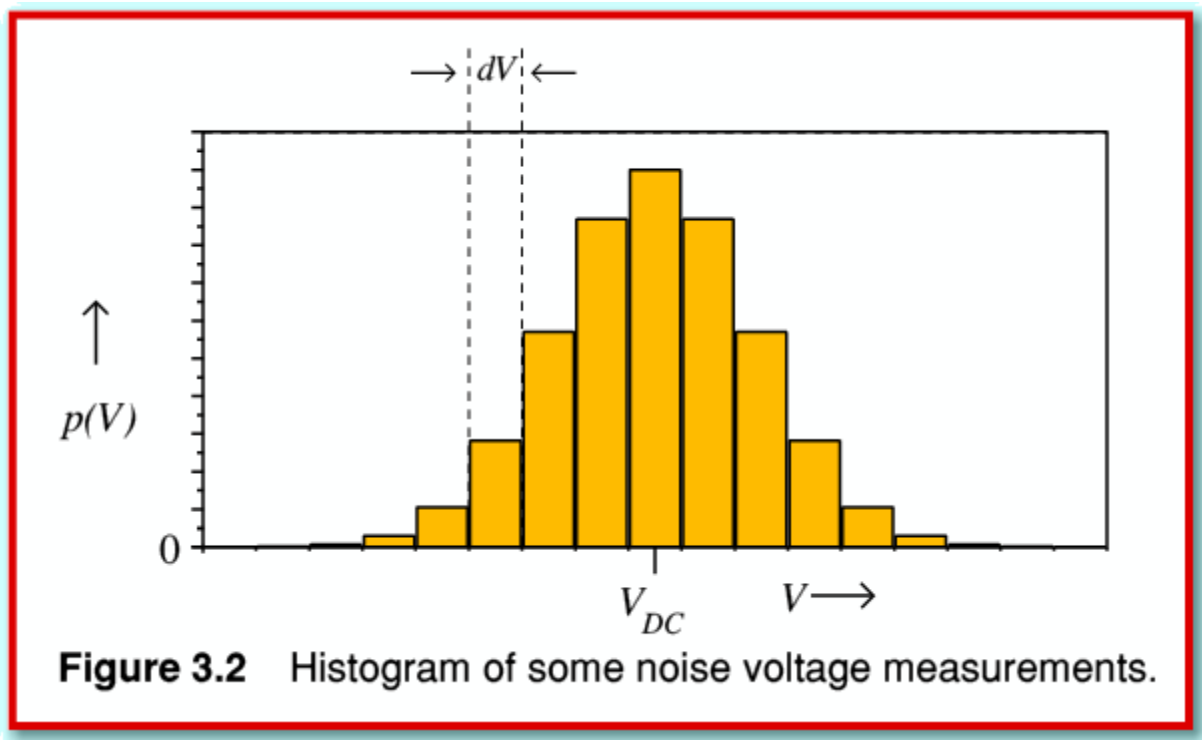
$$p\{V\} \cdot dV$$

that any future measurement of the voltage will give a result in any particular small range

$$V \rightarrow V + dV$$

This type of histogram is called a ***Probability Density Distribution*** of the fluctuations. From this two conclusions become apparent:

1) Firstly, the average of all the voltage measurements will be around zero volts. This isn't a surprise since there's no reason for the electrons to prefer to concentrate at one end of the resistor. For this reason, the average voltage won't tell us anything about how large the noise fluctuations are.



2) Secondly, the histogram will approximately fit what's called a ***Normal (or Gaussian)*** distribution of the form

$$p\{V\} \cdot dV \propto \text{Exp}\left\{\frac{-2V^2}{\sigma^2}\right\}$$

where σ is the “Standard Deviation”.

(This is in the limit of an infinite number of measurements) Small numbers of measurements won't generally show a nice Gaussian plot with its centre at zero) The value of σ which fits the observed distribution indicates how wide the distribution is, and is **measure of the amount of noise**.

The **standard deviation σ** is useful for theoretical reasons since the probability distribution is Gaussian. However it is more generally used to quantify a noise level in terms of an ***rms or root-mean-square*** quantity.

Imagine making a series of m measurements of the voltage (in this case)

$$v_1, v_2, \dots, v_j, \dots, v_m$$

We then calculate the rms voltage level which is defined as

$$v_{rms} \equiv \sqrt{\sum_{j=1}^m \frac{v_j^2}{m}}$$

In general we can simplify things by using the ‘angle brackets’, $\langle \rangle$, to indicate an averaged quantity. The expression above for the RMS becomes

$$v_{rms} = \sqrt{\langle v_j^2 \rangle}$$

$$v_j^2$$

Since v_j^2 will be always be positive or zero the RMS is always be positive whenever the Gaussian noise distribution has a width greater than zero.

The wider the distribution, the larger the rms voltage level.

Hence, unlike the mean voltage, the rms voltage is a useful indicator of the noise level.

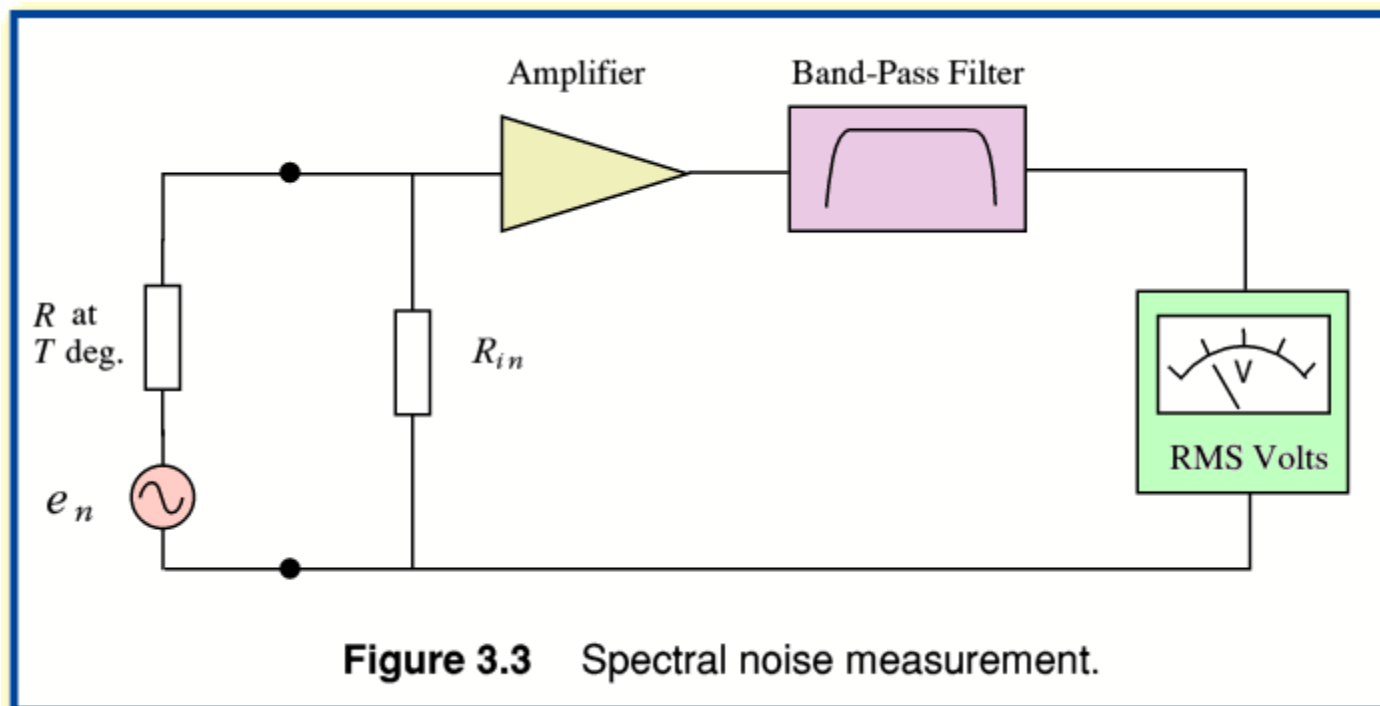
The rms voltage is of particular usefulness in practical situations because the amount of **power associated with a given voltage varies in proportion with the voltage squared**. Hence the average power level of some noise fluctuations can be expected to be proportional to

$$V_{rms}^2$$

Since thermal noise comes from thermal motions of the electrons **we can only get rid of it by cooling the resistor down to absolute zero** (which we cannot).

More generally, we expect the thermal noise level to vary in proportion with the temperature.

Up to now we looked at the statistical properties of noise in terms of its overall rms level and probability density function. Now we will show another way to think about Johnson noise.



As in figure 3.1 we're looking at the Johnson noise produced by a resistor. In this case the voltage fluctuations are amplified and passed through a *band-pass filter* to an rms voltmeter. **The filter only allows through frequencies in some range**

$$f_{min} < f < f_{max}$$

The filter is said to **pass a *bandwidth***

$$B = f_{max} - f_{min}$$

$$R_{in}$$

is the **input resistance of the amplifier**. Note that this diagram uses a common method of pretending that the noise generated in the resistor is actually coming from an invisible **random voltage generator**,

$$e_n$$

connected in series with an ‘**ideal**’ (i.e. **noise-free**) resistor. If we build a system like this we find that the rms fluctuations seen by the meter imply that the (imaginary) noise generator produces an average voltage-squared

$$\langle e_n^2 \rangle = 4kTB R$$

where: k is *Boltzmann's Constant* ($=1.38 \times 10^{-23}$ Ws/K); T is the resistor's temperature in Kelvin; R is its resistance in Ohms; and B is the bandwidth (in Hz) over which the noise voltage is observed.

In practice, the amplifier and all the other items in the circuit will also generate some noise. For now, however, we will assume that the amount of noise produced by R is large enough to swamp any other sources of random fluctuations. Applying Ohm's law to figure above we can say that the current entering the amplifier (i.e. flowing through R_{in}) must be

$$i = \frac{e_n}{(R + R_{in})}$$

The corresponding voltage seen at the amp's input (across R_{in}) will be

$$v = iR_{in} = \frac{e_n R_{in}}{(R + R_{in})}$$

hence the mean noise power entering the amplifier will be

$$N = \langle iv \rangle = \frac{\langle e_n^2 \rangle R_{in}}{(R + R_{in})^2}$$

For a given resistor, R , we can maximize this by taking the derivative with respect to R . We then find that the power is maximized when

$$R_{in} = R$$

This is sometimes called a **matched load**. If we calculate the power as above for this we get the *Maximum Available Noise Power*

$$P_{\max} = \langle e_n^2 \rangle / 4R$$

We can show from statistical mechanics that this system is equivalent to a single mode system and the power per mode in a system that is in thermal equilibrium is $\langle P_{\text{per mode}} \rangle = kT\beta$ or kTB where $\beta=B$ is the detection bandwidth.

If we view this system as being in thermal equilibrium we have

$$\langle P_{\text{per mode}} \rangle = kT\beta = P_{\max} = \langle e_n^2 \rangle / 4R$$

and hence

$$\langle e_n^2 \rangle = 4kTB R$$

This is the Johnson noise voltage **squared**.

The Johnson noise is thus $e_n = (4 kT\beta R)^{1/2}$

This represents the highest thermal noise power we can get to enter the amplifier's input terminals from the resistor. To achieve this we have to *match* the source and amplifier input resistances.

The *Noise Power Spectral Density* (NPSD) or just PSD at any frequency **is defined as the noise power in a 1 Hz bandwidth at that frequency.** Putting $B=1$ we can see that Johnson noise has a maximum available NPSD of just kT i.e. it only depends upon the absolute temperature and the value of Boltzmann's constant.

This means that Johnson noise has an NPSD which doesn't depend upon the fluctuation frequency. The same result is true of shot noise and many other forms of noise. Noise which has this character is said to be **White Noise** since we see the same power level in a fixed bandwidth at every frequency.

In practice no power spectrum can be truly **white** over an infinite frequency range. This is because the total power, integrated over the whole frequency range, would be infinite. In any real situation, the noise generating processes will be subject to some inherent mechanism which produces a finite noise

bandwidth. In practice real systems will only be able to respond to a small range of frequencies which is much smaller than the actual bandwidth of the noise being generated. This will limit any measured value for the total noise power.

For most purposes we can consider thermal and shot noise as ‘white’ over any frequency range of interest.

It is worth noting that electronic noise levels are often quoted in units of ***Volts per root Hertz or Amps per root Hertz.***

Common units for typical audio amplifiers are quoted in $\text{nV}/\sqrt{\text{Hz}}$ or $\text{pA}/\sqrt{\text{Hz}}$.

These figures are sometimes referred to as the NPSD though strictly speaking this is incorrect since power would be in $(\text{volts})^2$ or $(\text{amps})^2$.

This is because most measurement instruments are normally calibrated in terms of a voltage or current. For white noise we can expect the total noise level to be proportional to the measurement bandwidth.

The ‘odd’ units of NPSD's quoted per root Hertz is a reminder that a **noise level specified as an rms voltage or current** will increase with the square root of the measurement bandwidth.

Shot noise

Many forms of random process produce a Gaussian or Normal distribution of noise. Johnson noise occurs in all (non zero resistance) systems which aren't at absolute zero, hence it can't be avoided in normal electronics. Consider superconducting electronics that have $R=0$ however.

Another form of noise which is, in practice, unavoidable is ***Shot Noise***. As with thermal noise, this arises because of the **quantization of electrical charge** and hence you are **ultimately counting charges** and thus **subject to counting statistics** (remember voting and counting statistics).

Imagine a current flowing along a wire. While we often think of current as being the flow of positive charges it is almost always the flow of electrons and hence negative charges having charge $q \sim 1.6 \times 10^{-19}$ Coulombs.

To define the current we can imagine a surface through which the wire passes and count the number of charges, n , which cross the surface in a time, t . The

current, i , observed during each interval will then simply be given by

$$i = \frac{q n}{t}$$

One amp will thus be about **6 million trillion electrons per second (6×10^{18})!**

While there will be an average flow of electrons in one direction there will also be a substantial random component due to thermal (Brownian) motion. Each electron will have its own (almost) random velocity and separation from its neighbors.

When we repeatedly count the number of carriers passing thru a cross section of the wire in a given time interval in a series of m successive time intervals of equal duration, t , **we find that the counts will fluctuate randomly (we have to be very careful with this term)** from one interval to the next. Using these counts we can say that the typical (average) number of charges seen passing during each time t is

$$\langle n \rangle = \sum_{j=1}^m \frac{n_j}{m}$$

where n_j

is the number observed during the j th interval. The mean current flow observed during the measurement time will therefore be

$$I = \frac{\langle n \rangle q}{t}$$

During any specific time interval the observed current will be

$$i_j = \frac{n_j q}{t}$$

which will generally differ from the average I by an unpredictable amount.

The effect of these variations is therefore to make it appear that there is a randomly fluctuating noise current superimposed on the nominally steady current, I .

This is called **Shot Noise**.

The size of the current fluctuation

$$\Delta i_j$$

during each time period can be defined in terms of the variation in the numbers of charges passing in the period

$$\Delta n_j$$

i.e. we can say that

$$\Delta i_j = \frac{q \Delta n_j}{t} \quad \text{where} \quad \Delta n_j = n_j - \langle n \rangle$$

As with Johnson noise, we can make a large number of counts and determine the magnitude of the noise by making a statistical analysis of the results.

Once again we find that the resulting values have a Gaussian or *Normal* distribution in the limit of an infinite number of measurements.

By definition we can expect that

$$\langle \Delta n \rangle = 0$$

(since $\langle n \rangle$ is arranged to be the value which makes this true).

Hence, as with Johnson noise, we should use the **mean-squared variation, not the mean variation, as a measure of the amount of noise.** In this case,

taking many counts and performing a statistical analysis, we find that

$$\langle \Delta n^2 \rangle \approx \langle n \rangle$$

This is the result we discussed with counting or voting statistics.

Combining the above expressions we can link the magnitude of the current fluctuations to the mean current level and say that

$$\langle \Delta i^2 \rangle = \frac{q^2 \langle \Delta n^2 \rangle}{t^2} = \frac{q^2 \langle n \rangle}{t^2} = \frac{q^2}{t^2} \times \frac{I t}{q} = \frac{q I}{t}$$

Hence we find that the **mean square** of the random current fluctuations is approximately proportional to the average current while the **RMS** is proportional to the **square root of the current**. When done correctly with matching you get an increase of a factor of 2 so $\langle \Delta I^2 \rangle = 2 q I/t$. Since the current shot noise is normally quoted per root Hz (t=1 sec) for $\langle \Delta I^2 \rangle^{1/2}$ and per Hertz for $\langle \Delta I^2 \rangle$ and hence

$$\langle \Delta I^2 \rangle = 2 q I \text{ per Hz}$$

$$\langle \Delta I^2 \rangle^{1/2} = (2 q I)^{1/2} \text{ per root Hz}$$

Since some current and voltage is always necessary to carry a signal this noise is unavoidable (unless there's no signal) although we can reduce its level by reducing the magnitude of the signal current.

Here are some numbers for common currents

$\langle \Delta I^2 \rangle^{1/2} \sim 0.57$	nA/ $\sqrt{\text{Hz}}$	I= 1 amp	
$\langle \Delta I^2 \rangle^{1/2} \sim 18$	pA/ $\sqrt{\text{Hz}}$	I= 1 milli-amp	10^{-3}
$\langle \Delta I^2 \rangle^{1/2} \sim 0.57$	pA/ $\sqrt{\text{Hz}}$	I= 1 micro-amp	10^{-6}
$\langle \Delta I^2 \rangle^{1/2} \sim 18$	fA/ $\sqrt{\text{Hz}}$	I= 1 nano-amp	10^{-9}
$\langle \Delta I^2 \rangle^{1/2} \sim 0.57$	fA/ $\sqrt{\text{Hz}}$	I= 1 pico-amp	10^{-12}
$\langle \Delta I^2 \rangle^{1/2} \sim 18$	aA/ $\sqrt{\text{Hz}}$	I= 1 femto-amp	10^{-15}
$\langle \Delta I^2 \rangle^{1/2} \sim 0.57$	aA/ $\sqrt{\text{Hz}}$	I= 1 atto-amp	10^{-18}

Note the relative increase ($\Delta I/I$) as I becomes smaller. Why?

Other sorts of noise

A wide variety of physical processes produce noise. Some of these are similar to Johnson and shot noise in producing a flat noise spectrum. In other cases the noise level produced can be strongly frequency dependent. One of the most common form of frequency-dependent noise is called 1/f noise or sometimes referred to as “flicker noise”.

Unlike Johnson or shot noise which depend upon simple physical parameters (the temperature and current level respectively) 1/f noise is strongly dependent upon the details of the particular system. The term '1/f noise' covers a number of noise generating processes, some of which are poorly understood. For this form of noise the NPSD, S_n , varies with frequency approximately as

$$S_n \approx f^{-n}$$

where the value of the *index*, n , is typically around 1 but varies from case to case over a typical range of $0.5 < n < 2$.

As well as being widespread in electronic devices, random variations with a 1/f spectrum appear in processes as diverse as the traffic flow, the power spectrum of music and even in visual 2D art, the radio emissions from distant galaxies earthquakes ...

Some simple mathematically predictable (not random phase) '*Chaotic*' systems may produce this form of fluctuations.

Vacuum Techniques

Life sucks sometimes and so does this lab (it should).

Muon Physics