

# Calibration of the Far Infrared Absolute Spectrophotometer (*FIRAS*) for the Cosmic Background Explorer (*COBE*)<sup>1</sup> Satellite

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## ABSTRACT

The Far Infrared Absolute Spectrophotometer (FIRAS) is one of the three instruments on the Cosmic Background Explorer satellite, launched on November 18, 1989. The instrument compares the diffuse IR and microwave background radiation from the Big Bang origin of the universe against an accurate blackbody. The sensitivity and accuracy of the instrument are sufficient to detect deviations from blackbody form that are less than 1/1000 of the peak blackbody brightness, for each 5% spectral element and each  $7^\circ$  pixel. The overall spectral range is from 1 to  $100\text{ cm}^{-1}$ . The instrument and calibrator designs and procedures required to meet these goals are described.

# 1 INTRODUCTION

We describe the calibration algorithms and analyze the performance of the calibration reference blackbody for the Far Infrared Absolute Spectrophotometer (FIRAS) instrument on the Cosmic Background Explorer (COBE) satellite. The paper gives the instrument and calibrator designs, the detector gain compensation algorithm, the linear photometric algorithm, a discussion of photometric errors, the thermal accuracy requirement, a brief description of the beam profile determination, and an analysis of the performance of the calibrator itself.

The COBE (1, 2, 3, 4) was launched on November 18, 1989 on a Delta rocket, and carries two instruments to measure the spectrum and anisotropy of the the 2.7 K remnant of the Big Bang, as well as an instrument to measure the shorter wavelength diffuse IR background light of the universe. These instruments will make orders of magnitude improvements in the sensitivity and accuracy of the measurements.

The FIRAS spans the frequency range from  $\nu = 1$  to  $100 \text{ cm}^{-1}$ , has a spectral resolution  $\Delta\nu/\nu > 3\%$  limited by beam divergence, unapodized  $\Delta\nu > 0.1 \text{ cm}^{-1}$  limited by the maximum path difference of 5.6 cm, a circular beam profile  $7^\circ$  in diameter, and an rms sensitivity and accuracy specified as better than  $\nu I_\nu = 10^{-13} \text{ W/cm}^2\text{sr}$  for each resolution element in frequency and each  $7^\circ$  pixel on the sky. This sensitivity is approximately 0.1% of the peak brightness of a 2.7 K blackbody. Our calibrator design is estimated to be at least 10 times more accurate than this specification. The thermometric accuracy is specified as 0.001 K.

The FIRAS contains a Fourier transform spectrometer based on the Martin-Puplett (5) polarizing form of the Michelson interferometer. It is fully symmetrical, with complete separation of the two input and two output ports. This design has many advantages. It permits an extremely large étendue of  $1.5 \text{ cm}^2\text{sr}$ , and hence allows operation at frequencies as low as  $1 \text{ cm}^{-1}$ . It is a multiplexed instrument, allowing many wavelengths

to be measured simultaneously with the same detector. Its symmetrical design allows differential operation, instantaneously comparing the input with a reference input and modulating the difference. Its maximum optical efficiency is determined by the polarizers, which are quite effective up to frequencies of about  $1/4g$ , where  $g$  is the spacing of the polarizer wires. The FIRAS polarizers are made of 20 micrometer tungsten wires, plated with gold, and spaced on 53 micrometer centers, giving some response up to about  $100\text{ cm}^{-1}$ .

The radiation accepted from the sky is defined by a Winston cone, also known as a compound parabolic concentrator (6,7), and refocusses it with an elliptical cone. The cone, designated the Sky Horn, is connected to a flared section like a trumpet bell, which suppresses diffracted sidelobes over a very wide spectral band. A ray trace of the concentrator has been reported (8), and measurements of the sidelobes have been reported (9). According to these measurements, sidelobe response will be negligible except for the Moon when it is within about  $30^\circ$  of the line of sight.

Calibration of the instrument is made using a full beam external calibrator blackbody which can be moved into the aperture on command. The entire accuracy of the instrument is dependent on this calibrator, so most of this paper is devoted to a description of it and an analysis of its errors. The calibrator temperature is controlled by a thermometer - heater servo loop, and is stable within 0.001 K at 2.7 K. The temperature range is from 2 to 20 K and is monitored by 3 germanium resistance thermometers in a separate self calibrating AC ohmmeter circuit.

The shape of the calibrator is illustrated in Figure xx. It is shaped like a trumpet mute, with a central peak and a single groove, each with a full angle of  $\psi = 25^\circ$ . It is machined from two castings of Eccosorb (10) CR-110, one for the central peak, one for the remainder. This Eccosorb is an epoxy loaded with extremely fine iron powder, with a small admixture of Cab-o-sil, an even finer silica powder. The silica powder makes the

liquid epoxy thixotropic, so that the iron powder does not settle during the curing process. The optical properties of the Eccosorb have been reported (11 ,12 ,13) .

The calibrator is backed by corrugated high purity copper sheets, with saw cuts across the corrugations to give flexibility in both directions. The copper is glued to the Eccosorb using more Eccosorb as adhesive, taking care to have enough adhesive to penetrate the saw cuts in the copper but not to cover the copper. This design allows the differential thermal contraction of the two materials to occur without exceeding the bond strength. Over the outside of the copper is placed an aluminum foil cap, since the Eccosorb is not entirely opaque and there are gaps in the coverage of the copper sheets. The outside of this structure is covered with a multilayer insulation blanket, containing xx layers of aluminized Kapton separated by layers of Dacron net. This insulation is required because a portion of the calibrator is exposed to infrared emission from warm portions of the spacecraft.

In addition, the second input of the instrument is filled by a temperature controlled blackbody at the input of a small version of the Sky Horn. This Internal Reference is adjusted to null the signal from the prime input, reducing the dynamic range of the instrument by at least a factor of 100. The temperatures of both concentrators are also controlled and commandable from 2 to 20 K. The reference input concentrator and blackbody produce the same étendue and focusing as the primary input concentrator.

The entire instrument is operated in a vacuum, and is cooled to a temperature of about 1.5 K by conduction to a superfluid liquid helium tank. The cryostat is essentially identical to that used in the Infrared Astronomical Satellite (IRAS), and was manufactured by the Ball Aerospace Division. The satellite spins at 0.8 rpm about its symmetry axis to satisfy the requirements of the other two instruments, and the FIRAS line of sight is along the spin axis. The orbit is circular, at an

altitude of 900 km, and is inclined  $99^\circ$  to the equator, with a node crossing time of 6 am and 6 pm. The spin axis of the COBE is maintained  $94.5^\circ$  away from the Sun, and points approximately to the zenith. A large conical shield protects the cryostat and instruments from direct radiation from the Sun and the Earth. The Sun never illuminates the instruments or cryostat, but the COBE orbit inclination combined with the inclination of the Earth's equator to the ecliptic do allow the Earth limb to rise a few degrees above the plane of the instrument and sunshade apertures during about 1/6 of the orbit for 1/4 of the year. The edge of the shield is coplanar with the entrance apertures of the cryogenic instruments, so there is no geometrical view factor for radiation from the shield itself into the instruments.

## 2 DETECTOR GAIN COMPENSATION ALGORITHM

The FIRAS detectors do not have constant gains or time constants because they are not temperature controlled and because they can be used under large signal conditions. They are not temperature controlled for simplicity of construction and reliability, and because temperature control would require raising the temperature and sacrificing sensitivity. There is thermal crosstalk in the instrument, in that variations of the calibrator and beam defining concentrator temperatures, the mirror scanning length, and the operation of the other instrument in the cryostat all change the detector temperatures quite substantially. Over the range of temperature from 1.5 K to 2.5 K the detector gains change by a factor of xx. This gain variation must be compensated before the photometric calibration can be done.

The detectors for the FIRAS are large area composite bolometers (14) , constructed of a diamond octagon 7.8 mm across and 25 micrometers thick blackened with a coating of chrome-gold having a surface resistance

of about  $267 \Omega/\text{square}$ . The temperature of the diamond is determined with a silicon resistance thermometer having approximately  $10 \text{ M}\Omega$  resistance, and is biased with DC current through a cryogenic wirewound load resistor of  $40 \text{ M}\Omega$ . There are 4 detectors, two on each output of the interferometer. These two view beams separated by a dichroic filter having a split at  $20 \text{ cm}^{-1}$ . The high frequency detectors have about 7 msec time constants, and two low frequency ones about 50 msec time constants. Sensitivities are about  $5 \times 10^{-15} \text{ W/Hz}^{1/2}$  for the low frequency detectors, and  $1.5 \times 10^{-14}$  for the faster ones.

The detectors are described by equations giving the heat capacity, electrical resistance, and conducted thermal power as a function of temperature. In addition, it has been found that the electrical resistance is also a function of the applied electric field. In all of these cases, a simple functional form with a few parameters describes the properties well, so a detector gain model may be constructed and fitted to the detector data (15 ,16 ,17) . These equations are:

$$R(T, E) = R_0 \exp(T_0/T)^{1/2}(x / \sinh(x)), (1)$$

$$x = eE\xi/ak_B T, (2)$$

$$W(T, T_c) = W_0(T^{\beta+1} - T_c^{\beta+1}), (3)$$

$$C(T) = C_0 T^\gamma, (4)$$

$$E = E_b - IZ_L, (5)$$

$$E = R(T, E)I, (6)$$

$$C(T)dT/dt = Q + EI - W(T, T_c). (7)$$

In these equations,  $R$  is the detector resistance,  $R_0$  is a constant giving the detector resistance at infinite temperature,  $T_0$  is a characteristic temperature for the detector resistance function,  $x$  is a dimensionless variable expressing the degree of nonideality of the detector resistance,  $E$

is the voltage across the detector,  $e$  is the electron charge,  $\xi$  is a characteristic hopping length for carriers in the detector element,  $a$  is the length of the detector,  $k_B$  is Boltzmann's constant, and  $T$  is the temperature of the detector. The function  $W$  gives the power conducted out of the bolometer element by the leads,  $W_0$  is a constant,  $T_c$  is the detector housing temperature,  $\beta$  is expected to be about 1 for metallic leads,  $C$  is the heat capacity,  $C_0$  is a constant, and  $\gamma$  is a constant expected to be about 3 for a nonmetallic crystal.  $E_b$  is the bias supply voltage,  $I$  is the detector current, and  $Z_L$  is the load impedance. The differential equation is simply the thermal balance equation, where  $Q$  is the absorbed radiant power to be determined.

The DC equilibrium detector voltage was measured as a function of temperature and current, giving the static parameters. Heat capacities were determined from the electrical transfer function, measured from an AC input voltage applied at the load resistor to the detector output voltage. Fitting accuracies are typically 1% for the low frequency detectors and 3% for the high frequency detectors, which have larger nonideal effects (values of  $x$ ). An improved fit might be obtainable using a more complicated function for the nonideal resistance, such as replacing  $(x/\sinh(x))$  with  $(x(1+a)/(\sinh(x) + \sinh(ax)))$ . This allows for more than one characteristic hopping length scale.

The detector model is used for data analysis in two ways. The first and simplest way is to compute the small signal parameters  $S_0$  and  $\tau_e$ , so that the frequency dependent responsivity is

$$S(\omega) = S_0/(1 + j\omega\tau_e). \quad (8)$$

The incoming interferograms are Fourier transformed and corrected for this computed responsivity in the frequency domain. The values of  $S_0$  and  $\tau_e$  are determined from the commanded detector current, the measured DC voltage on the detector, and the measured detector housing temperature.



The second method is a direct solution of the differential equation for the detector, and is valid for large signals as well. The signals can become large when observing the Moon or the calibrators when they are hot. The maximum signal the instrument can record is 0.1 V AC, which is not small relative to the bias voltages of 0.4 V for the low frequency and 1.4 V for the high frequency detectors.

The data stream gives us the voltage  $E$ , and the bias voltage on the load resistor and the temperature  $T_c$  are also known. The voltage  $E$  is composed of two parts, a DC voltage and an AC voltage which are telemetered separately and then added in the computer. From  $E$ , we compute  $I$  from the detector load resistance and the bias supply voltage. From  $E$  and  $I$  we compute the detector temperature  $T$ , from the full nonideal expression for the resistance. From  $T$  we compute  $W$  and  $C$ , and the derivative  $dT/dt$  is computed with a Fourier transform method to guarantee agreement with the small signal formulas. The remaining variable is  $Q$  which is now deduced.

The accuracy required of the detector gain algorithm is a few percent, given by the ultimate accuracy desired and the signal suppression achieved by the differential operation of the instrument. At the low frequency end of the range (1 to 20  $\text{cm}^{-1}$ ), the sky is expected to approximate a blackbody, so that an effective null can be achieved with the internal reference body temperature at about 2.7 K. Moreover, the calibration at these low frequencies can be achieved without raising the calibrators to high temperatures and changing the detector temperatures greatly.

On the other hand, the high frequency portion of the range requires relatively warm blackbody calibrators and hence large thermal crosstalk. The validity of the detector algorithm and parameters will be verified by the linearity of the calibration of detector spectral output versus blackbody brightness at each frequency. The Moon will be observed frequently, and its spectrum is already known. Jupiter will also be observed, but is barely detectable.

### 3 CALIBRATION ALGORITHM

#### 3.1 Linear analysis

The photometric calibration is assumed to be linear after correction for the detector gain variations. The calibration method implements the following formula:

$$I_\nu = G(\nu)[D(\nu) + \sum_i \epsilon_i(\nu) B_\nu(T_i)], \quad (9)$$

where  $I_\nu$  is the complex Fourier transformed interferogram for spatial frequency  $\nu$ ,  $G(\nu)$  is the complex gain function of the interferometer, including factors for the detector, preamplifier, analog filtering and amplification, and digital prefilters,  $D(\nu)$  is a constant photometric offset arising from unmeasured causes, the summation  $\sum_i$  is over the possible sources of radiation entering the instrument,  $\epsilon_i(\nu)$  is the effective emissivity of each such source relative to the external blackbody calibrator, and  $B_\nu(T_i)$  is the Planck blackbody function for the temperature of the  $i$ th source. We have included terms for the external calibrator (for which  $\epsilon = 1$ ), the internal reference body (for which  $\epsilon \approx -1$ ), the "sky horn" parabolic concentrator (for which  $\epsilon \propto (\nu/1 \text{ cm}^{-1})^{1/2}$ , and the equivalent for the reference side with  $\epsilon \approx -\epsilon$  (sky horn). The negative effective emissivities result from the differential nature of the instrument. We have not been able to measure a separate term corresponding to the temperature of the interferometer structure, since this temperature does not change very much in normal operation, and its effects are included in  $D(\nu)$ .

The aim of the calibration is to measure the functions  $G(\nu)$ ,  $D(\nu)$ , and  $\epsilon_i(\nu)$ . The function  $G(\nu)$  is determined from measurements of differences of spectra taken with different temperatures of the external calibrator, since its  $\epsilon$  is assumed to be identically unity. The functions  $D(\nu)$  and  $\epsilon_i(\nu)$  are determined by varying the temperatures of all the controllable bodies and making a least squares fit. We have the option of giving different

weights to different temperature combinations, so we can force an accurate determination of  $D(\nu)$  at the standard operating condition. This standard operating condition will have all controllable bodies at the same temperature as the nominal cosmic background radiation temperature, approximately 2.7 K.

Initial determinations of the calibration functions are shown in Figs xx. They are derived from a limited amount of preflight data, and will be replaced with inflight calibrations after launch. Such recalibration is essential since the instrument alignment may change during the launch vibration. The function  $G(\nu)$ , combined with the measured noise levels from the detectors, may be used to predict the ultimate sensitivity of the measurements. These predicted sensitivities are shown in Figs xx. These sensitivities meet the requirements of the Project.

The photometric linearity assumption is only approximately correct, and the following known nonlinear effects will be investigated from the flight and ground calibration data. In addition, the detector gain model may not represent the detector perfectly.

### 3.2 Subharmonic Responses

Spurious harmonic and subharmonic responses have been neglected in the calibration algorithm, since measurements show that they are small, less than 1%. A subharmonic half-frequency response can occur in the FIRAS because both mirrors move instead of only one, and interference can occur between waves scattered from fixed structures and waves travelling the normal paths. This is not expected to be important for wideband incoherent sources, but it has been observed with coherent microwave oscillator sources. Effects of such subharmonic response would be seen in the calibration process as deviations on the Rayleigh-Jeans side of the peak.

### 3.3 Harmonic Responses

Harmonic responses can occur from waves making multiple passes through the FIRAS. These are potentially important because the aperture stops are reflective concentrator cones rather than black baffles. They have also been observed with microwave oscillators, but not with far IR laser inputs. These effects depend in detail on the ray positions and angles, since the nominal design calls for perfect coupling between the sky and reference blackbody to the detectors. Ray traces show that about 20% of the light from the inputs does not reach the detectors because of aberrations (xx), and a similar fraction of light reflected from the detector region may be returned to the interferometer by the input cone. Hence, one may expect that a few percent of the light reaching the detectors has been modulated 3 times by the interferometer. Effects of such harmonic response would be seen in the calibration process using blackbody sources, as deviations from linearity on the Wien side of the peak.

### 3.4 Baseline Curvature

The FIRAS interferometer has such a large étendue and beam divergence that its transmission efficiency depends significantly on the positions of the reflecting mirrors, changing by about 1/2 % over the stroke. As a result, the total power reaching both sets of detectors is not quite constant as a function of path difference, and each interferogram appears to be built on a curved baseline. The Fourier transform of this curvature has most of its energy at very low frequencies. In effect, each input frequency produces a spectral ghost at very low frequencies. To remove these effects, a fourth order polynomial is fitted to the interferogram and subtracted prior to further processing. This is equivalent to a digital high pass filter.

### 3.5 Improper phases

Our method assumes that  $G(\nu)$  can be complex but that the  $\epsilon_i(\nu)$  are real. This is not necessarily accurate, since the interferometer is not perfectly efficient or ideal (18) . The method can be generalized by allowing the  $\epsilon_i(\nu)$  to be complex, in which case our linear fitting

procedure will determine them with their phases. Our phase functions are determined from the external calibrator response, and the same phases are used for all interferograms, so linearity is preserved.

## 4 EFFECTS OF VIBRATION ON CALIBRATION

Vibration produces changes to the photometric calibration of the instrument as well as producing spectrometer ghosts. These vibrations produce phase errors because the servo loop driving the mirrors does not have infinite bandwidth, nor do the electronics for the sampling circuits or the detector. We use a fixed phase function to phase correct all interferograms, so linearity is preserved in the presence of vibration. The effective interferogram resulting from a vibrating instrument is then

$$I(x) = \int \cos(2\pi\nu x + \phi(x))S(\nu)d\nu, \quad (10)$$

where  $\phi(x) = 2\pi\nu\Delta x(x)$  is the random phase error function depending on the path difference  $x$ ,  $\Delta x(x)$  is the position error of the sampling at position  $x$ , and  $S(\nu)$  is the detected spectrum. We may expand this to get

$$I(x) = \int [\cos(2\pi\nu x) \cos(\phi(x)) - \sin(2\pi\nu x) \sin(\phi)]S(\nu)d\nu, \quad (11)$$

which may be further expanded for small  $\phi$  to give

$$I(x) \approx \int S(\nu)d\nu \cos(2\pi\nu x)(1 - \phi^2/2) + \Delta x(x)(dI(x)/dx) \quad (12)$$

In the presence of a random vibration environment, and when a large number of interferograms are being averaged together, we seek the average effects of the variable  $\phi$ . We see that the first term  $(1 - \phi^2/2)$  gives a reduction of the gain of the system, according to the mean square phase error introduced. Errors introduced by this gain change are proportional to the signal level, which will be minimized by operating the

instrument near null. If the vibration level is constant during the entire mission, then its effects will be absorbed in the calibration factors.

The second term gives a random noise term which has zero mean, and does not bias the resulting average interferogram. This extra noise term will be minimized in the FIRAS operation by nulling the interferogram to first order, except of course during calibration sequences. In addition, the interferograms will be compared with one another before they are averaged together, and those that appear significantly different from a representative template will be rejected from the average. The statistical analysis of the noise resulting from this process will be derived from comparisons of individual spectra to each other and to the mean. Since the largest errors occur near the interferogram peaks, the noises in the spectra will be strongly correlated from one frequency to another, invalidating the usual expectations about smoothing a spectrum to improve the noise level.

## **5 THERMAL ACCURACY REQUIREMENT**

The calibrator is designed to have no temperature gradient, by preventing heat flows through it. The entire calibrator is supported from a copper ring whose temperature is controlled, and this copper ring is attached to the support arm. To confirm that there are no temperature gradients, germanium resistance thermometers are installed on the ring and near the tip.

The temperature gradient must be less than a few milliKelvin to assure that the thermometers read a suitable average temperature. However, for the purposes of comparing the spectrum with an ideal best fit blackbody, the temperature gradient can be relaxed. An average of Planck functions at a variety of temperatures is a Planck function of the average

temperature, except for second order effects. These effects are proportional to the second derivative of the Planck function with respect to temperature:

$$\langle B_\nu \rangle = B_\nu(T_m) + (1/2) \langle (T - T_m)^2 \rangle (d^2 B_\nu / dT^2), \quad (13)$$

$$T_m = \langle T \rangle, \quad (14)$$

where  $T$  is the temperature, and  $T_m$  is the mean temperature. We define  $x = h\nu c / k_B T$ , where  $h$  is Planck's constant,  $\nu$  is the spatial frequency  $1/\lambda$ ,  $k_B$  is Boltzmann's constant, and  $\eta$  is the photon mode occupation number. The second derivative may be evaluated as

$$(d^2 B_\nu / dT^2) = (2hc^2\nu^3 / T^2) x e^x (x + 2 - 2e^x + x e^x) / (e^x - 1)^3. \quad (15)$$

As an example, at  $T = 2.7$ , we find that  $(1/2)\nu(d^2 B / dT^2)$  is maximum at  $\nu = 12 \text{ cm}^{-1}$ , and has the value  $8.01 \times 10^{-11} \text{ W/cm}^2 \text{ sr K}^2$ . With the requirement that  $\nu B$  be accurate to  $10^{-14} \text{ W/cm}^2 \text{ sr}$ , we find that the rms temperature variation must be less than  $11.2 \times 10^{-3} \text{ K}$ , 4 parts in 1000. Conversely, if the cosmic background radiation is a mixture of blackbody fields with this rms variation, the spectrum will be distorted at the  $10^{-14} \text{ W/cm}^2 \text{ sr}$  level.

## 6 BEAM PROFILE DETERMINATION

The beam profile will be measured using the Moon as a source. However, accurate beam profiles are not required for the primary purpose of the instrument, which is measurement of a nearly isotropic brightness. The reason is that the calibration is derived from a full beam blackbody calibrator, which simulates the geometry of the isotropic sky quite precisely. The beam profile is only required for computing the brightness of an observed source which does not overfill the beam.

Approximate beam profiles were measured with coherent microwave radiation at 32, 53, and 90 GHz, with a laser at 118 micrometers, and with near IR and visible broadband radiation. They confirm that the beam profile is close to the geometrical optics prediction at short wavelengths, a 7° diameter circular beam. Sidelobes are described by surface scattering formulae at short wavelengths, and by the Geometrical Theory of Diffraction at long wavelengths. The main forward beam profile deviates greatly from a circular top hat at long wavelengths where only a few diffraction modes are accepted, at least when measured with a single mode source and detector.

## 7 CALIBRATOR ANALYSIS

The external calibrator is the ultimate limit on the accuracy of the instrument. It must produce the same radiation field that a perfect blackbody filling the beam defining concentrator would produce. It must have an accurately measured and uniform temperature, it must not leak around the edge, it must not reflect or transmit radiation from significantly hotter or colder regions, and it must respond to temperature controller commands. We have designed it to have a spectrum which is blackbody within  $10^{-14}$  W/cm<sup>2</sup>sr in terms of  $\nu I_\nu$  at a temperature of 2.7 K.

To achieve thermometric accuracy of 0.001 K, we calibrated our flight germanium resistance thermometers against transfer standard thermometers from the National Bureau of Standards. To guarantee preservation of this accuracy we read the thermometers with self calibrating 14 bit AC (40 Hz) ohmmeters, and have provided 3 of them on two separate ohmmeters.

### 7.1 CALIBRATOR AS PART OF A CAVITY



The calibrator body forms a part of a cavity composed of four essential parts: the calibrator (12 cm diameter), a highly reflective compound parabolic concentrator (also 12 cm diameter, tapering to a small aperture), the small aperture (7.8 mm diameter) leading to the spectrometer, and the gap between the calibrator and the concentrator. In this case, the most important source of radiation impinging on the calibrator is the calibrator body, by way of reflection from the concentrator.

The next most important source is the concentrator, whose direct emissivity toward the spectrometer is determined in the calibration method described above. The radiation from the concentrator which reflects from the calibrator would be an important error source if its temperature were unknown, but fortunately it can be controlled and made equal to the calibrator temperature. In that case, the calibrator and the concentrator form a large isothermal cavity, whose radiation is very nearly blackbody. The remaining errors arise from radiation entering from the spectrometer at the small aperture, and from leakage around the gap between calibrator and concentrator.

## 7.2 CALIBRATOR REFLECTANCE VIEWED FROM SPECTROMETER

The spectrometer is a source of radiation, which can be measured directly, or reflected from the calibrator and then measured. The spectrometer structure and optical elements are composed primarily of aluminum. The parts outside the optical beam are black anodized, but this treatment has little effect on the far IR properties. The spectrometer is not perfectly isothermal, since there are heat flows in the structure originating in the detector preamplifiers, the motor that drives the mirrors, and the heaters that control the concentrator and calibrator temperatures. Most of the radiation produced by this structure and optics cannot be modulated by the interferometer, since it does not enter with the correct direction through the input polarizer and cannot be split coherently by the

beamsplitter. Any radiation which can be modulated will be measured in the calibration process and incorporated in the photometric offset function  $D(\nu)$ . As long as this radiation is a constant, or is correlated with some measured temperature, it does not cause a photometric error.

An error can arise, however, when the emission from the spectrometer enters the cavity of the combined calibrator and concentrator, reflects from the calibrator, and returns to the spectrometer. This radiation would not return if the calibrator were removed from the concentrator, so it cannot be determined in the calibration procedure. The beam of radiation incident on the calibrator fills the aperture nearly uniformly, and has nearly the same angular distribution as the beam profile, a circle  $7^\circ$  in diameter. Most of this incident beam is absorbed on its first bounce on the calibrator, so higher order paths are negligible.

Only a small portion of the beam incident on the calibrator can scatter back into the ray bundle accepted by the concentrator and the spectrometer. For comparison, if the calibrator were a flat Lambertian scatterer with total reflectance  $R_{LS}$ , the fraction of the incident beam accepted by the spectrometer would be  $f = R_{LS}\Omega/\pi$ , where the solid angle of the accepted beam is  $\Omega = \pi \sin^2(\theta/2)$ , and  $\theta = 7^\circ$ . Numerically,  $\Omega/\pi = 3.7 \times 10^{-3}$ .

### 7.2.1 DIFFUSE SURFACE REFLECTANCE

The surface texture appears similar to that of a machined metal surface having a surface roughness of  $\sigma = 5$  micrometers rms. We approximate the calibrator diffuse reflectance by

$$R_{surf} = 4R_n(\Omega/\pi) \sin(\psi/2)(\sigma k)^2, \quad (16)$$

where  $k = 2\pi\nu = 2\pi/\lambda$  is the wavevector,  $R_n \approx 0.1$  is the normal reflectance of a polished surface,  $\psi = 25^\circ$  is the full angle of the cone and groove, and the sine function accounts for the angle of incidence of radiation from the spectrometer. Evaluating at a wavelength of 1 mm, we

find  $R_{surf} = 3.2 \times 10^{-7}$ , quite negligible, showing that a more exact calculation is unnecessary.

### 7.2.2 BACK SURFACE DIFFUSE REFLECTANCE

The Eccosorb is not thick enough to be entirely opaque, so the back surface is partly visible through it. The back surface is partly covered with irregularly shaped copper foils. We take its surface reflectance to be unity with an angular distribution which would appear Lambertian if viewed through a completely transparent Eccosorb. We think this is conservative, in the sense that much of the scattered radiation would be trapped inside the dielectric. Based on the formulas given above, we should have

$$R_{back} = (\Omega/\pi)T_s^2 e^{-2\alpha t}, \quad (17)$$

where  $T_s = 1 - R_s$  is the transmittance of the front surface,  $\alpha$  is the measured absorption coefficient of the material, and  $t$  is the thickness (1.27 cm). Cryogenic measurements showed that  $\alpha \approx 0.3 + 0.45 \nu$  over the range of frequencies used here, and the surface reflectance is  $R_s \approx 0.08 + 0.06/\nu$ , where  $\nu$  is measured in reciprocal centimeters. Evaluating this at  $\nu = 1 \text{ cm}^{-1}$ , we find  $R_{back} = 4.1 \times 10^{-4}$ . This number is not negligible, but decreases exponentially as the frequency increases.

### 7.2.3 SPECULAR SURFACE REFLECTANCES

To estimate the specular reflectance of the calibrator, we approximate it by a V groove of the same included angle in an infinitely thick medium. In this case, there is no mixing of polarizations, and all the angles of incidence are known. A ray originating in the spectrometer will be reflected 7 times before exiting the V groove, at angles from normal of 77.5, 52.5, 27.5, 2.5, 22.5, 47.5, and 72.5 degrees. We estimate the refractive index from the normal surface reflectance  $R_s$  and use the Fresnel formulas to compute all the reflectances. Averaging over polarizations gives  $R_{spec} = 5 \times 10^{-5}$  at  $1 \text{ cm}^{-1}$ .

Only a fraction of the returned specular beam is directed back toward the spectrometer. The horn defines a circular field of view of  $7^\circ$  diameter, so one may consider that it sends a circular bundle of rays toward the V groove. The circle comes back shifted over by  $5^\circ$  because of the accumulated effect of the 7 reflections, so that it overlaps slightly with the circle representing the rays that can be received by the instrument. The fraction of the area that overlaps is computed as 14%. In other words, if the calibrator were a V groove of two mirrors, 14% of the beam originating in the spectrometer would return to it to be detected. Since the actual calibrator is not a simple V groove, the overlap fraction could be either smaller or larger, but the computation is not precise enough to merit detailed attention.

A better estimate of the surface reflections would include the back surface reflectance of the material as well. We approximate the back surface by a smooth metal coating. The net reflectance is then

$$R_{net} = |r_s + t_s^2 w / (1 - w)|^2, \quad (18)$$

where

$$w = e^{-\alpha t \sec(\theta)} e^{4\pi i \nu \cos(\theta)} \quad (19)$$

is the propagator for a round trip through the Eccosorb,  $r_s^2 = R_s$ ,  $t_s^2 = T_s$ ,  $n$  is the refractive index,  $\nu$  is the spatial frequency, and  $\theta$  is the angle from normal inside the material. The result of this computation is shown in Figure xx, and is also not negligible at long wavelengths.

## 7.2.4 DIFFRACTION FROM EDGE AND GROOVE

The V groove and the calibrator edge are both sources of diffraction which can scatter light back to the spectrometer. We apply the Geometric Theory of Diffraction (19) (GTD) to the edge scattering as follows. We have not attempted a full solution of the scattering problem for the groove, which would involve a complex process of tracing many rays as

they bounce off the groove faces, or solving the boundary value problem for a groove with absorbing faces. Instead, we argue that an absorbing groove should not scatter more strongly than a ridge, since the essential thing is that there is a discontinuity in the boundary conditions for the waves at the groove vertex. Aside from this discontinuity, the groove should behave as in geometrical optics.

The outer edge of the calibrator is also an important scatterer. It is not directly visible (according to geometrical optics) from the spectrometer because of the flare of the concentrator, but at low frequencies, diffraction around the flare permits coupling. Computations from the GTD show that the flare produces almost no attenuation at the calibrator edge for frequencies less than  $50 \text{ cm}^{-1}$ . The reason is that the edge is not close enough to the surface of the flare, nor far enough around the curve. We expect a clearance of approximately 0.06 cm, which seems necessary to prevent jamming the calibrator into the concentrator.

The groove and edge both produce specular beams, in the sense that the diffracted waves all add up in phase when the angle of scatter is the same as the angle of incidence with respect to the plane of the circular scattering edge. However, there is not an especially large amount of the total energy in this central peak. The diffracted radiation field is estimated as follows. We start with Sommerfeld's solution (20) for diffraction at a straight edge of an opaque screen. We approximate the groove or the ridge at the edge of the calibrator as a semi-infinite metallic hollow cylinder illuminated from the open end, as shown in Fig xx. Sommerfeld gives his solution as

$$u = ((1 + i)e^{ikr}/4(\pi kr)^{1/2})(\sec(\phi - \alpha)/2 \pm \sec(\phi + \alpha)/2), \quad (20)$$

where the incident field is unity,  $r$  is the distance from the edge,  $k = 2\pi/\lambda$ ,  $\psi$  is the angle from the screen surface to the outgoing ray, and  $\alpha$  is the angle from screen surface to incoming ray. In this geometry, both  $\phi$  and  $\alpha$  are  $180^\circ$  so the first secant is 1 and the second is -1. The  $\pm$  sign

must be chosen according to polarization state, being plus when the electric field is perpendicular to the edge and - when parallel. As expected, one of the two polarizations (the case where the electric field is perpendicular to the scattering edge) does not diffract in the backward direction. The edge has an approximately isotropic scattering behavior in the backward direction, with a scattering effective width of about  $\pi r |u|^2 = \lambda/8\pi$  after averaging over polarizations.

To continue, we use the Geometrical Theory of Diffraction to modify the straight edge diffraction formula to apply to a curved edge. Following the usual prescription, the rays diffracted from the curve appear to diverge from a caustic point or line a distance  $r_0$  away from the scattering edge. For incident rays parallel to the axis of the circle, the caustic is on the axis of the circle and is located  $r_0 = r_1 \sin \theta$  away from the circle, so the diffracted amplitude is multiplied by the factor  $(r_0/(r + r_0))^{1/2}$ . This factor expresses the conservation of energy in a family of rays crossing through the caustic points or lines. In the far field limit, fans of rays from two opposite points on the circle reach the same angle off axis and interfere, giving

$$I(\theta) = |v_p r|^2 = (\sec(\phi - \alpha)/2 \pm \sec(\phi + \alpha)/2)^2 \times (r_1/2\pi k \sin \theta)(\cos^2(z)), \quad (21)$$

where  $I(\theta)$  is the scattering function with respect to solid angle,  $r_1$  is the radius of the scattering ring,  $\theta$  is the scattering angle with respect to the axis, and  $z = kr_1 \sin \theta$  describes the diffraction pattern. The singularity of  $I(\theta)$  at  $\theta = 0$  is not genuine, but is integrable over solid angle even in this approximation.

To see how to evaluate at small angles, we recognize that the scattering function should have a factor representing the interference of wavelets from different points of the bright ring. It should be of the form

$$\int e^{ikr} d\phi = \int e^{ikr_1 \sin \theta \cos \phi} d\phi = J_0(z)$$

$$\approx (2/\pi z)^{1/2} \cos(z - \pi/4). \quad (22)$$

Substituting this form in place of the most similar factors in the previous expression for  $I(\theta)$  we find that

$$I(\theta) \approx (\sec(\phi - \alpha)/2 \pm \sec(\phi + \alpha)/2)^2 (r_1^2/4) |J_0(z)|^2. \quad (23)$$

To get an approximate integral over solid angle, we replace  $\cos^2$  by  $1/2$ , neglect the variations of the secant functions, and average over the polarizations. The maximum value of  $z$  is  $kr_1$  and the solid angle integral is then

$$\begin{aligned} I_{tot} &= \int 2\pi I(\theta) \sin \theta d\theta \approx \lambda(2\pi r_1)/(2\pi)^2 \\ &= A_{scat}. \end{aligned} \quad (24)$$

We use this equation to define the effective scattering area of the groove or edge,  $A_{scat}$ . In other words, the equivalent scattering width of the edge is only  $\lambda/(2\pi)^2$ .

The nearly specular portion of the diffractive backscatter allows the spectrometer to see itself. For this we evaluate the solid angle integral out to  $\theta = 3.5^\circ$  off axis, giving a smaller value of

$$R_{diff} = A_{spec}/A_{cal} \approx \lambda \theta_{max}/2\pi^2 r_1. \quad (25)$$

However, all this area couples the spectrometer emission back to the spectrometer. It contributes a direct loss of emissivity of the calibrator body, equal to  $R_{diff}$ , which for  $\lambda = 1$  cm is  $5 \times 10^{-4}$ . An estimate for this effect is plotted in Fig. xx.

### 7.2.5 DIFFRACTION FROM THE POINT

Diffraction from the central peak of the calibrator is negligible and has not been evaluated in detail. From dimensional arguments, one expects the total scattering area of the peak to be proportional to the square of

the characteristic length of the the wave variation,  $\lambda/2\pi = 1/k$ , since this is the only length available from which to construct something having the dimensions of area. Comparing this to the calibrator area,  $A_{pk}/A_{cal} \approx 1/\pi(kr_1)^2$ , where  $r_1$  is the radius of the calibrator, about 6 cm. At a frequency of  $1 \text{ cm}^{-1}$ , this ratio is  $2.2 \times 10^{-4}$ , and it decreases rapidly as the frequency increases. This would not be detectable by our instrument. On the basis of comparison to scattering at an edge, we also expect to find that there is a dimensionless factor multiplying all these numbers, with a value much less than one.

### 7.3 LEAKAGE AROUND EDGE

There is a gap between the calibrator and the concentrator to guarantee free movement, about  $g_1 = 0.06 \text{ cm}$  wide, which has an total area of 2% of the calibrator area. This gap is sealed by two rings of aluminized Kapton fingers, embedded in the Eccosorb and pressing against the concentrator, as illustrated in Fig. xx. Each ring appears to make a perfect seal, but for the purposes of estimation we assume that each transmits 3% of the incident radiation. In that case, the effective area of the gap is reduced to

$$f_{gap} = 1.8 \times 10^{-5} \quad (26)$$

of the total area of the calibrator.

In addition, the metallic covering of the calibrator does not extend all the way to the edge. It leaves an exposed area about  $g_2 = 0.06 \text{ cm}$  wide, which permits radiation to enter the Eccosorb directly and be transmitted into the spectrometer. The fractional area associated with this leak is then

$$f_{leak} = (2g_2/r)T_s^2 e^{-\alpha t}, \quad (27)$$

where the length  $t$  is about 2/3 of the total thickness of the calibrator, or 0.8 cm. At 1 cm wavelength, we estimate  $f = 8 \times 10^{-3}$ , but direct microwave measurements show that the actual response is much smaller than this, approximately xx. For the curves in Figure xx, we have used



this normalization and extrapolated to other wavelengths using the attenuation coefficient in the material. Fortunately, there are no very bright sources in the field of view at 1 cm wavelength.

Both of these leakage paths will be measured in the flight cryostat, using emission from the warm cryostat cover as a source. Its temperature can be varied over the range from 10 K to 150 K by flowing cold cryogen through pipes, and the resulting spectra will be measured. The emissivity of the cover will be measured as well, by removing the calibrator and observing the cover directly.

The important sources of radiation incident on the gap between calibrator and horn in flight are the sky, the multilayer insulation blanket on the outside of the calibrator, and other warm portions of the spacecraft.

### 7.3.1 SKY

The sky approximates a 2.7 K blackbody at long wavelengths, so when the calibrator is at the sky temperature, the leakage radiation does not affect the total calibrator brightness. At shorter wavelengths, there are two very bright sources in the sky, the Galactic plane and the Moon. The Moon has been observed with the calibrator in place and leakage will be determined directly.

### 7.3.2 MULTILAYER INSULATION (MLI) BLANKET

The MLI blanket protects the calibrator from xx milliwatts of incident heat from the warm sunshade and warm parts of the cryostat. In the process, it heats up to a temperature of about 60 K in order to reradiate the heat it absorbs. It is also a source of radiation in the passband of the FIRAS, illuminating the gap between calibrator and concentrator. The incident intensity can be estimated as:

$$I_\nu = B_\nu(T_b) \times (0.003\nu^{1/2} \sec(\zeta) \times \Delta\zeta, (28)$$

where  $\zeta$  is the angle between the normal to the MLI surface and the direction to the gap. The emissivity of the metal surface is proportional to the square root of the frequency, as for standard bulk metal, and the secant function expresses the variation of emissivity with angle. In this case,  $\zeta \approx 90^\circ$ , so  $\Delta\zeta \sec(\zeta) \approx 1$ , and we obtain the same effective emissivity as if the MLI filled the field of view at normal incidence. Evaluating at  $\nu = 100 \text{ cm}^{-1}$ ,  $\nu I_\nu = 6.4 \times 10^{-14} \text{ W/cm}^2\text{sr}$ .

### 7.3.3 OTHER WARM SURFACES

The same surfaces that radiate toward the calibrator and warm up the insulation also emit in the instrument passband. However, tests of the view factor from those directions into the region of the gap between calibrator and concentrator showed that the attenuation was very large, approximately xx. As a result, the MLI emission is the dominant error source.

## 8 SUMMARY AND CONCLUSIONS

The calibration methods for the Far Infrared Absolute Spectrophotometer (FIRAS) have been described and the accuracy estimated. The calibrator accuracy is sufficient to meet the design requirement of  $\Delta(\nu I_\nu) < 10^{-14} \text{ W/cm}^2\text{sr}$ , about a part in  $10^4$  of the peak brightness of a 2.7 K blackbody.

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## 10 TABLES

## 11 FIGURE CAPTIONS

## 12 REFERENCES

1. J.C. Mather, "The Cosmic Background Explorer (COBE), Optical Engineering, 21, 769 (1982)
2. C. Bauer, "COBE's Quest", Astronomy, xx, 74, (Aug. 1986)
3. J.C. Mather, "A look at the primeval explosion", New Scientist, xx, 48 (16 Jan 1986)
4. J.C. Mather, "Capabilities of the Cosmic Background Explorer", in 13th Texas Symposium on Relativistic Astrophysics, Edxx, (pubxx, cityxx, 1987)
5. D.H. Martin and E. Puplett, "titlxxx", Infrared Phys., 10, 57 (1970)

6. W.T. Welford and R. Winston, The Optics of Nonimaging Concentrators, (Academic Press, New York, 1978).
7. J.C. Mather, "Broad-Band Flared Horn with Low Sidelobes", IEEE Trans. Antennas and Propagation, AP-29, 967, (1981)
8. M.S. Miller, W.L. Eichhorn, and J.C. Mather, "Sky input horn for a far-infrared interferometer", Optics Ltrs, 7, (1982)
9. J.C. Mather, M. Toral, H. Hemmati, "Heat trap with flare as multimode antenna", Appl. Optics, 25, 2826, (1986)
10. Emerson and Cuming, Canton, Mass., "High Loss Dielectric Microwave Absorbers, Tech. Bull. 2-6, Rev. 1980
11. H. Hemmati, J.C. Mather, W.L. Eichhorn, "Submillimeter and millimeter wave characterization of absorbing materials", Appl. Optics, 24, 4489, (1985).
12. J.B. Peterson and P.L. Richards, "A Cryogenic Blackbody for Millimeter Wavelengths", Int. J. Infrared and Millimeter Waves, 5, 1507 (1984)
13. J.B. Peterson, P.L. Richards, and T. Timusk, "Spectrum of the Cosmic Background at Millimeter Wavelengths", Phys. Rev. Lett., 55, 332 (1985)
14. A. Serlemitsos, "titlexx", Proc. SPIE xxx, xxx (1988)
15. J.C. Mather, "Bolometer Noise: Nonequilibrium Theory", Appl. Opt. 21,1125 (1982)
16. J.C. Mather, "Bolometers: Ultimate Sensitivity, Optimization, and Amplifier Coupling", Appl. Opt. 23, 584 (1984)
17. J.C. Mather, "Electrical self-calibration of nonideal bolometers",

Appl. Opt. 23, 3181 (1984)

18. H.E. Revercomb, H. Buijs, H.B. Howell, D.D. LaPorte, W.L. Smith, and L.A. Sromovsky, "Radiometric calibration of IR Fourier transform spectrometers: solution to a problem with the High-Resolution Interferometer Sounder", Appl. Opt. 27, 3210 (1988)

19. B.R. Levy and J.B. Keller, "Diffraction by a smooth object," Comm. Pure Appl. Math, XII,159 (1959)

20. A. Sommerfeld, Optics, (Academic Press, New York, 1964), p. 261, equation 21.