PHYSICAL REVIEW LETTERS

VOLUME 66 | 29 APRIL 1991 | NUMBER 17

Cosmic-Structure Constraints from a One-Degree Microwave-Background Anisotropy Experiment

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(Received 8 May 1990; revised manuscript received 21 December 1990)

A recent anisotropy experiment with a 30′ beam is close to optimal for testing models in which cosmic structure arose from inflation-generated Gaussian density perturbations. Using a Bayesian analysis to constrain the amplitude of the perturbation spectrum, we show that adiabatic hot-dark-matter models are convincingly ruled out and cold-dark-matter models have anisotropies near our derived limits. Theories with broken scale invariance such as isocurvature baryon models are strongly constrained, as are models with extra power crafted to give significant large-scale structures and flows.

PACS numbers: 98.70.Vc, 98.80.Cq

In this Letter we show that the South Pole experiment of Lubin and co-workers,[1] which samples scales of $\approx 1°$, is nearly optimal for testing theories with scale-invariant initial fluctuations and can be used to set important new constraints on models of cosmic-structure formation. The microwave-background pattern can be expanded in multipole moments $\Delta T(\hat{\mathbf{q}})/T = \sum_{lm} a_{lm} Y_{lm}(\hat{\mathbf{q}})$, where $\hat{\mathbf{q}}$ is the angular direction of the incoming photons. If the fluctuations are Gaussian [as assumed in cold-dark-matter (CDM) models], the multipole coefficients a_{lm} are Gaussian distributed with zero mean and variance $C_l = \langle |a_{lm}|^2 \rangle$, with $\langle a^{\dagger}_{l'm'} a_{lm} \rangle = 0$ for $l \neq l'$, $m \neq m'$. In Fig. 1 we show the angular power spectra C_l for a number of theoretical models computed using the techniques of Refs. 2 and 3.

While the shape of C_l is determined by the shape of the initial (e.g., postinflation) spectrum there is an arbitrary overall normalization amplitude. It has now become conventional to define a biasing factor b_ρ to characterize this amplitude,[6] which is unity if mass traces light and greater than unity if galaxies are more clustered than the mass distribution. The constraints we derive below are all expressed in terms of b_ρ. We characterize the shape of the fluctuation spectrum by a local power-law index $n_s = d\ln\langle|\delta_{\rm in}(k)|^2\rangle/d\ln k$ for the initial density fluctuations $\delta_{\rm in}(k)$. The only well-motivated theoretical models start with pure scale-invariant initial conditions, with $n_s = 1$ for adiabatic perturbations and $n_s = -3$ for isocurvature perturbations. It is possible, however, to produce power spectra with ramps, mountains, and other shapes in some models of fluctuation generation;[7] even for these, constant n_s is often a reasonable approximation since we need only assume that n_s is slowly varying from the normalization scale, $\sim 5h^{-1}$ Mpc, to the wave-number region probed by the South Pole experiment [$k^{-1} \approx (20\text{–}80)\Omega^{-1/2}h^{-1}$ Mpc is the half-power region].

The effectiveness of an experiment in probing the temperature power spectrum C_l can be simply characterized by a filter function W_l, in terms of which the theoretical rms variance $\langle(\Delta T/T)^2_{\rm expt}\rangle$ for the experiment is $\sum_l W_l(2l+2)C_l/4\pi$. In Fig. 1, spectra ($l^2C_l/2\pi$, power per $\ln l$ for large l) are compared with the filter functions of several recent experiments. Note that the South Pole experiment is particularly well matched to CDM-type models, among others.

The experimental arrangement of Lubin and co-workers[1] is most easily described in polar coordinates

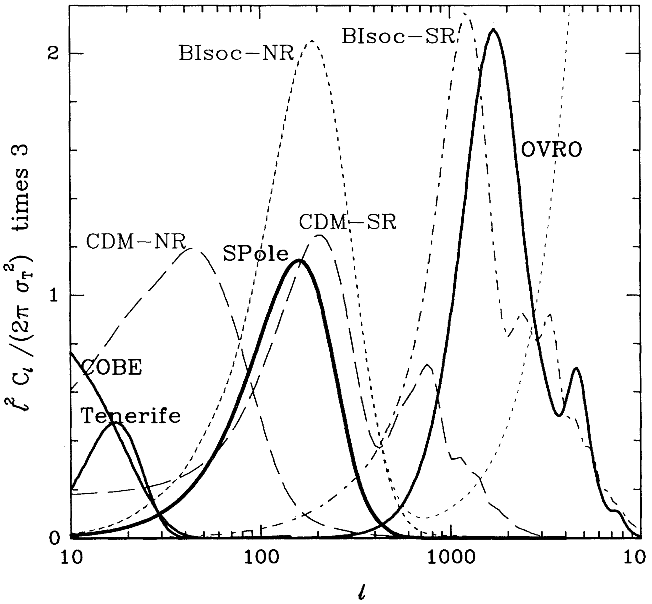

FIG. 1. Filter functions W_l (heavy solid curves) for the South Pole experiment, the OVRO (Ref. 4) and Tenerife (Ref. 5) experiments, and the COBE differential-microwave-radiometer experiment (Ref. 16). Also plotted are angular power spectra $l^2C_l/2\pi$ normalized by their integral σ_T^2 (and multiplied by a factor of 3 for plotting purposes). For a fixed experimental configuration, the filters just slide along in l space as the beam size changes. An optimal experiment is one for which the product of the filter and the power has the largest integral. Thus, the multipole range of the South Pole experiment is ideal for optimizing the signal from models like CDM with standard recombination (SR, rightmost long-dashed curve, with $b_\rho\sigma_T=3.5\times10^{-5}$) and low-$\Omega$ no-recombination (NR) isocurvature baryon models (an NR $n_s=0$, $\Omega=\Omega_B=0.2$ model is the short-dashed curve, with $b_\rho\sigma_T=2.0\times10^{-5}$). Low-$\Omega$ isocurvature baryon models with SR, are, on the other hand, best probed by experiments such as OVRO (an SR $n_s=0$, $\Omega=\Omega_B=0.2$ model is the dot-dashed curve, with $b_\rho\sigma_T=9.5\times10^{-5}$). $\Omega=1$ models with NR such as the leftmost long-dashed CDM model, with $b_\rho\sigma_T=2.5\times10^{-5}$, are best probed at larger ($\sim3°$) angles. However, it is unlikely in CDM models that gas can have formed stars early enough to wipe out small-angle anisotropies by reionization (Ref. 2). Including nonlinear effects in the $n_s=0$ NR isocurvature baryon model adds a great deal of short-distance power, raising $b_\rho\sigma_T$ to $\approx10^{-4}$ (short-dashed curve rising at large l).

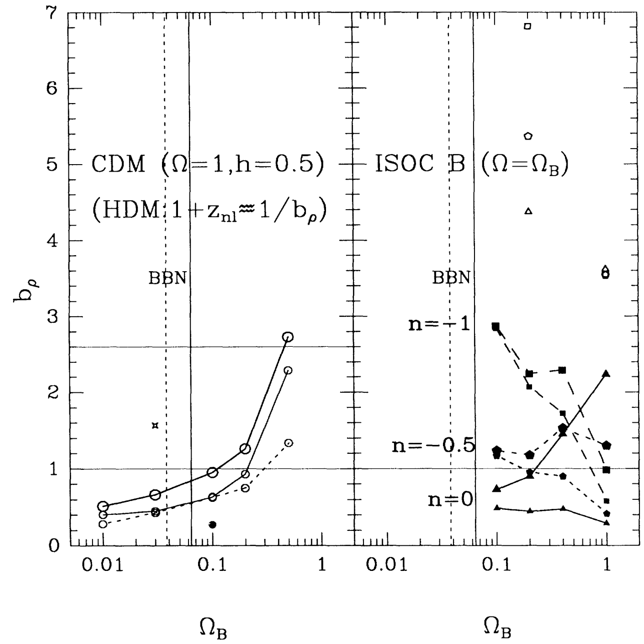

FIG. 2. The 95%-credible-limit biasing factor $b_{\rho,95}$ for $\Omega=1$ CDM models (left panel) and $\Omega=\Omega_B\leq1$ isocurvature baryon models (right panel) as a function of Ω_B; lower values are disallowed by this C.L. criterion. In both panels, open symbols denote SR models, and closed denote NR models. All models shown have $h=0.5$. The region between the lines at 1.0 and 2.6 indicate the range of b_ρ deemed likely for the CDM model from clustering and velocity data. The dotted curve is for the higher-sensitivity but nonoptimal OVRO experiment alone, next is the South Pole limit, and the topmost curve is for the combined data. The star denotes an isocurvature axion model (Ref. 9). These constraints also apply to HDM models (with the relation to the nonlinear redshift z_{nl} shown). The vertical lines denote an upper bound (solid line) from standard big-bang nucleosynthesis (Ref. 10) (BBN) and a less convincing lower bound (dashed line). In the right panel, triangles denote $n_s=0$ isocurvature baryon models; pentagons, $n_s=-0.5$ models; and squares, $n_s=-1$ models. These limits are derived using both the South Pole and OVRO data. The upper curves for each n_s include nonlinear effects (Refs. 3 and 11) (which the OVRO experiment is sensitive to), while the lower curves are from linear theory only, and are approximately what we get for the South Pole experiment alone. $b_\rho=1$ is usually adopted for isocurvature baryon models.

(θ,φ) centered on the South Pole. The beam was kept at constant zenith angle $\theta_z=17°$, but swept back and forth through $\sim2^h$ of right ascension, stopping to take data at $N_D=9$ patches separated by $\theta_{sep}=1°$ on the sky, with right ascension $\bar\varphi_j=\varphi_V+[j-(N_D+1)/2]\varphi_*$, $j=1,\ldots,N_D$, where $\varphi_*=3.4°$ and $\varphi_V=323°$. At each patch j, the beam of size $\theta_{FWHM}\approx30'$ performed an oscillation of amplitude $\phi_A\sin\theta_z=0.7°$ on the sky with a frequency $\nu=10$ Hz, so the instantaneous right ascension was $\varphi_j(t)=\bar\varphi_j+\phi_A\sin(2\pi\nu t)$. The signal was multiplied by 1 on half of the time cycle and -1 on the other half, with the switch occurring when φ_j passed through $\bar\varphi_j$. The data were obtained by sweeping across the nine patches, spending about a minute per patch, then sweeping back, with about 250 half sweeps for 72 h of data; on each half sweep, the gradient and average were removed from the data. We model the signal as a true time-independent cosmic signal plus a time-dependent (and therefore angle-dependent) instrumental noise plus an atmospheric noise with both time and angular dependence.[1] Thus, an unknown amount of gradient and average were removed from the signal in the final co-added data set. The data points have Gaussian-distributed errors with an $N_D \times N_D$ (diagonal) correlation matrix C_{Dij} about the observed values Δ_{Dj}, averaged over a cycle.

We model the observed signal as the difference $\Delta_{Dj} = \Delta_{Tj} - g_j$ of a true sky signal derived from the theory, Δ_{Tj}, and an average and gradient of unknown amplitude, $g_j = \alpha u_{0j} + \beta u_{1j}$, where $u_{0j} \equiv 1$ and $u_{1j} \equiv \bar{\varphi}_j$, for $j = 1, \ldots, N_D$. The theoretical signal Δ_{Tj} is Gaussian distributed with a correlation matrix $C_{Tij} = \langle \Delta_{Ti} \Delta_{Tj} \rangle$ between patches i and j given by

$$C_{Tij} = \sum_l U^{ij}_{\rm osc}(l)(2l+1)C_l/4\pi, \quad U^{ij}_{\rm osc}(l) = \sum\nolimits_m \mathcal{F}_l^2 [Y_{lm}(\theta_z,0)]^2 \cos[m(\varphi_j - \varphi_i)] 4[H_0(m\phi_A)]^2, \tag{1}$$

where $\mathcal{F}_l$ describes the beam profile, which is approximately $\mathcal{F}_l \approx \exp[-(l\theta_s)^2/2]$ (where $\theta_s = 0.425\theta_{\rm FWHM}$), and $H_0(z)$ is the Struve function of index 0.

We parametrize the theory in terms of the biasing factor ($C_l \propto b_\rho^{-2}$), assuming fixed shapes for C_l (which do depend upon the theory under consideration). We use Bayesian methods[8] to confront the theories with the data. The Bayes theorem gives the three-parameter probability $P(b_\rho^{-1},\alpha,\beta|D,E)\,db_\rho^{-1}\,d\alpha\,d\beta$ of the theory being correct given the new data D by

$$P(b_\rho^{-1},\alpha,\beta|D,E) = \mathcal{L} P(b_\rho^{-1}|E) P(\alpha,\beta|E)/P(D|E)\,,$$

where E denotes prior information (including assumptions about the theory, e.g, that it is a CDM theory with Gaussian initial conditions) and $P(D|E)$ is a normalization factor. The likelihood function is $\mathcal{L} = P(D|b_\rho^{-1},\alpha,\beta,E)$. Since α and β are not separately determined in the experiment, we integrate over the unknown average and gradient to obtain the one-parameter (marginal) distribution in terms of a modified likelihood function $\mathcal{L}'$:

$$\begin{aligned} &P(b_\rho^{-1}|D,E)\,db_\rho^{-1} \propto \mathcal{L}' P(b_\rho^{-1}|E)\,db_\rho^{-1}, \quad \ln\mathcal{L}' = -\chi^2/2 - \tfrac{1}{2}\ln[(2\pi)^{(N_D-2)}\det(C_t)\det(\tau)]\,, \\ &\chi^2 = \sum_{ij} \bar{\Delta}_{Di}\bar{\Delta}_{Dj} \left[(C_t^{-1})_{ij} - \sum_{\mu,\nu=0,1} \sum_{i',j'} (C_t^{-1})_{ii'} u_{\mu i'} (\tau^{-1})_{\mu\nu} u_{\nu j'} (C_t^{-1})_{j'j} \right], \\ &C_t \equiv C_D + C_T, \quad \tau_{\mu\nu} = \sum_{i'j'} u_{\mu i'} (C_t^{-1})_{i'j'} u_{\nu j'}, \quad \mu,\nu = 0,1\,. \end{aligned} \tag{2}$$

To obtain (2), we adopted the recommended[8] noninformative prior for α and β, namely, one that is uniform in both. We feel this is the most conservative assumption in the absence of further experimental monitoring, which could ultimately lead to a fully deterministic model of these systematics. The proportionality constant is found by ensuring that integration over b_ρ^{-1} gives unity. We assume the prior $P(b_\rho^{-1})$ in (2) is constant, with the restriction that $b_\rho^{-1} \geq 0$ since $b_\rho^{-1} = 0$ corresponds to no signal; this seems relatively natural and is conservative in the sense that upper limits derived from other priors {e.g., the noninformative prior $P(b_\rho^{-1}) \propto b_\rho^{-1}$] typically have higher values of b_ρ. Although values of $b_\rho^{-1} \gg 1$ are ruled out by dynamical observations, the likelihood function falls off so rapidly that an upper cutoff on b_ρ^{-1} makes no difference in practice. In Bayesian analysis, one defines[8] the 95% *credible level* (C.L.) $b_{\rho,95}^{-1}$ to be that value for which $P(b_\rho^{-1} < b_{\rho,95}^{-1}|D,E) = 0.95$.

Figure 2 shows the constraints on b_ρ for CDM and isocurvature baryon models as a function of Ω_B (the cosmological density in baryons) using the 95%-C.L. criterion. For the CDM models, we take $\Omega = 1$, $n_s = 1$, $h = 0.5$, and express our constraints on b_ρ as a function of Ω_B. To explain the clustering of galaxies and large-scale velocity flows, values of b_ρ in the range 1–2.6 have been proposed. CDM models with $\Omega < 1$ or with a nonzero cosmological constant are more strongly constrained as are isocurvature CDM models.[9] However, if reionization occurs early, the constraint is not as strong, as is shown for the $\Omega = 1$, $\Omega_B = 0.1$ CDM no-recombination (NR) model (solid circle in the left panel in Fig. 2).

We can also use the CDM constraints for massive-neutrino [hot-dark-matter (HDM)] models, which have angular power spectra similar to those for CDM. HDM models must be antibiased, $b_\rho < 1$, to ensure that galaxies form early enough. We can characterize this by the redshift $z_{\rm nl}$ at which the linear fluctuations reach an rms value of unity, related to b_ρ by $1 + z_{\rm nl} \approx 1.1 b_\rho^{-1}$. Thus for the $h = 0.5$, $\Omega_B = 0.1$ ($m_\nu \approx 22$ eV) model, $z_{\rm nl} \lesssim 0$ gives the constraint from the combined data, a strong limit independent of assumptions about where galaxies actually form in HDM theories.

The isocurvature baryon models with initial spectral index $n_s \approx -1$ that were popular in the 1970s have been resurrected recently.[12] Since there is a great deal of power at short distances in these models, star formation is expected to occur very early; hence it seems likely that the Universe would have been reionized (via photoionization) shortly after the usual epoch of recombination at $z \sim 1000$. Our NR models therefore provide a more realistic description of the anisotropies expected in isocurvature baryon models, though our limits can be modified slightly if one allows arbitrary freedom in the ionization history.[13] In reionized models a significant anisotropy is generated on arcminute scales, from quadratic nonlinearities in the scattering; this effect[3,11] is included in the upper curves for each n_s shown in Fig. 2.

If we assume $b_\rho=1$, most of the $n_s \lesssim 0$ NR models are ruled out, leaving only a small allowed region at $n_s \sim 0$ around $\Omega_B \sim 0.1$–0.2; for standard-recombination (SR) models even this region is disallowed. The simplest isocurvature baryon model compatible with inflation, i.e., $\Omega=1$ and $n_s=-3$, is *very* strongly ruled out. However, to be compatible with conventional primordial nucleosynthesis, $0.038 \lesssim \Omega_B \lesssim 0.064$ is required for $h=0.5$ (Ref. 10) and the residual $1-\Omega_B$ would have to be made up with vacuum energy for these models to be compatible with inflation. The constraints are then less stringent, although the $n_s=-3$ flat cases would still be strongly ruled out.

For an adiabatic $n_s=1$ (scale-invariant) spectrum, the power per $\ln k$ in large-scale gravitational-potential fluctuations is[7] $d\sigma_\Phi^2/d\ln k \approx (2\times 10^{-5}/b_\rho)^2$. Assuming the analogous scale-invariant form[2] for C_l on large angles, RELICT 1 reached[14] $b_\rho > 0.4$ at the 95% C.L., which should soon be surpassed by COBE. The value we obtain, $b_\rho \gtrsim 0.8$, is tantalizingly closer to the predictions of the standard CDM model. Modest improvements expected in the near future in intermediate-angle anisotropy experiments should either rule out or verify this theory. With the current limits there is little room for the extra fluctuation power often invoked to explain the large-scale structure data.[7,15]

P.M.L. and P.R.M. were supported by the NSF under Grant No. DPP878-15985 and NSF's Center for Particle Astrophysics, by NASA under Grants No. NAGW-1062 and No. GRSP NGT-50192, and by the California Space Institute.

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