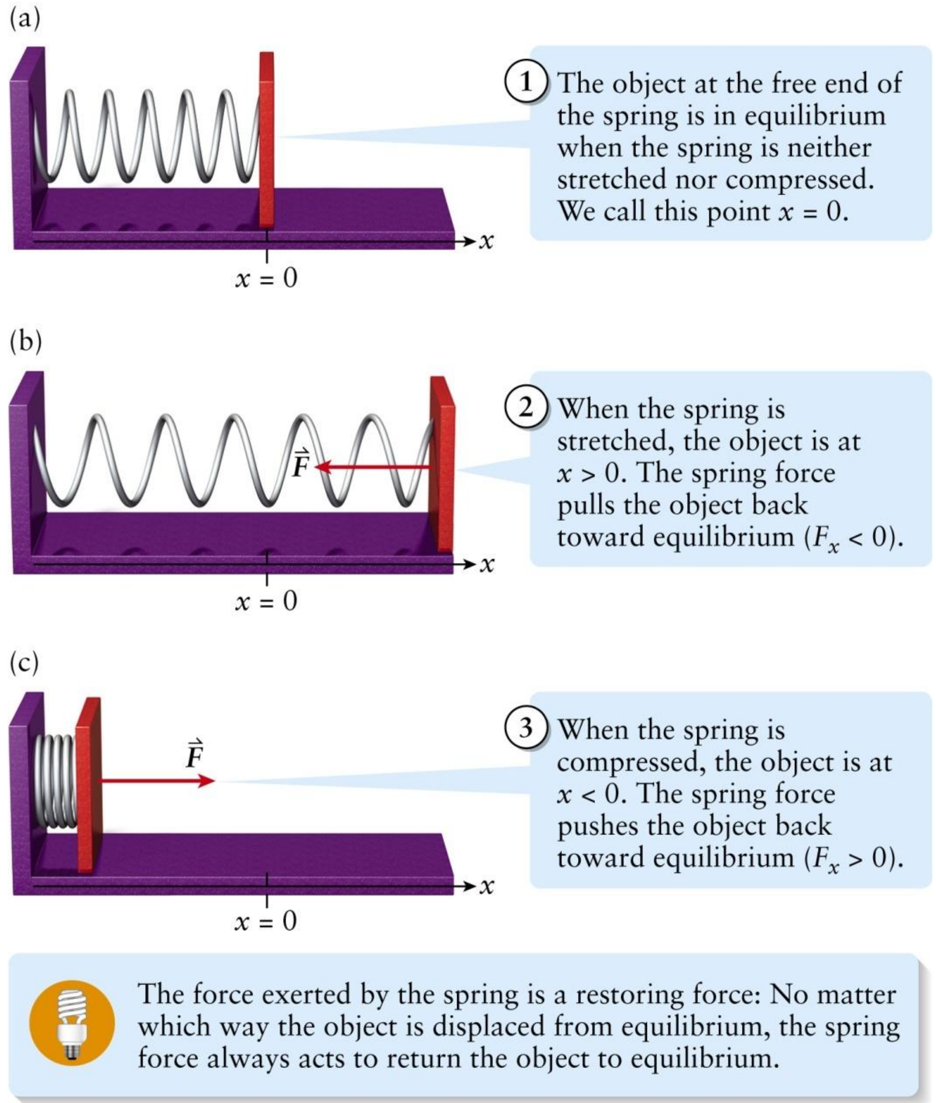
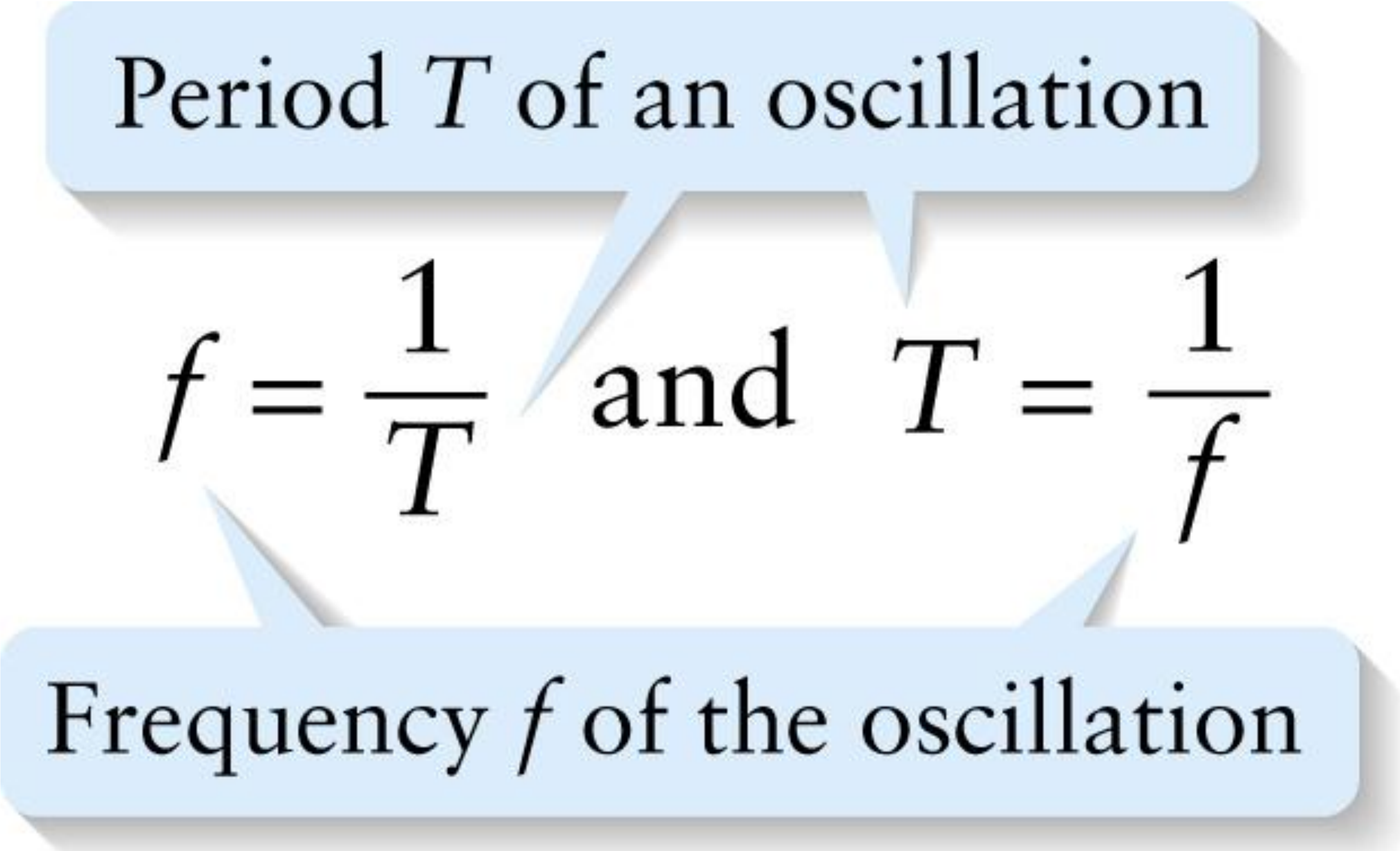
Chapter 12
Oscillations



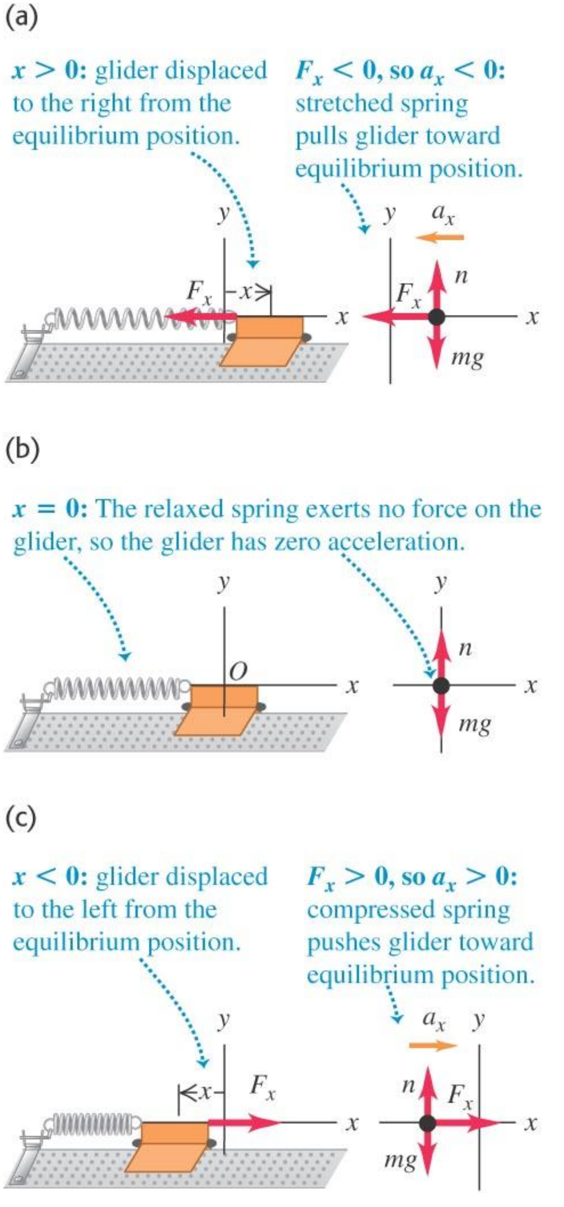
(a)
1 The object at the free end of the spring is in equilibrium when the spring is neither stretched nor compressed. We call this point $x = 0$.
x
$x = 0$
(b)
$\vec{F}$
2 When the spring is stretched, the object is at $x > 0$. The spring force pulls the object back toward equilibrium ($F_x < 0$).
x
$x = 0$
(c)
$\vec{F}$
3 When the spring is compressed, the object is at $x < 0$. The spring force pushes the object back toward equilibrium ($F_x > 0$).
x
$x = 0$
The force exerted by the spring is a restoring force: No matter which way the object is displaced from equilibrium, the spring force always acts to return the object to equilibrium.

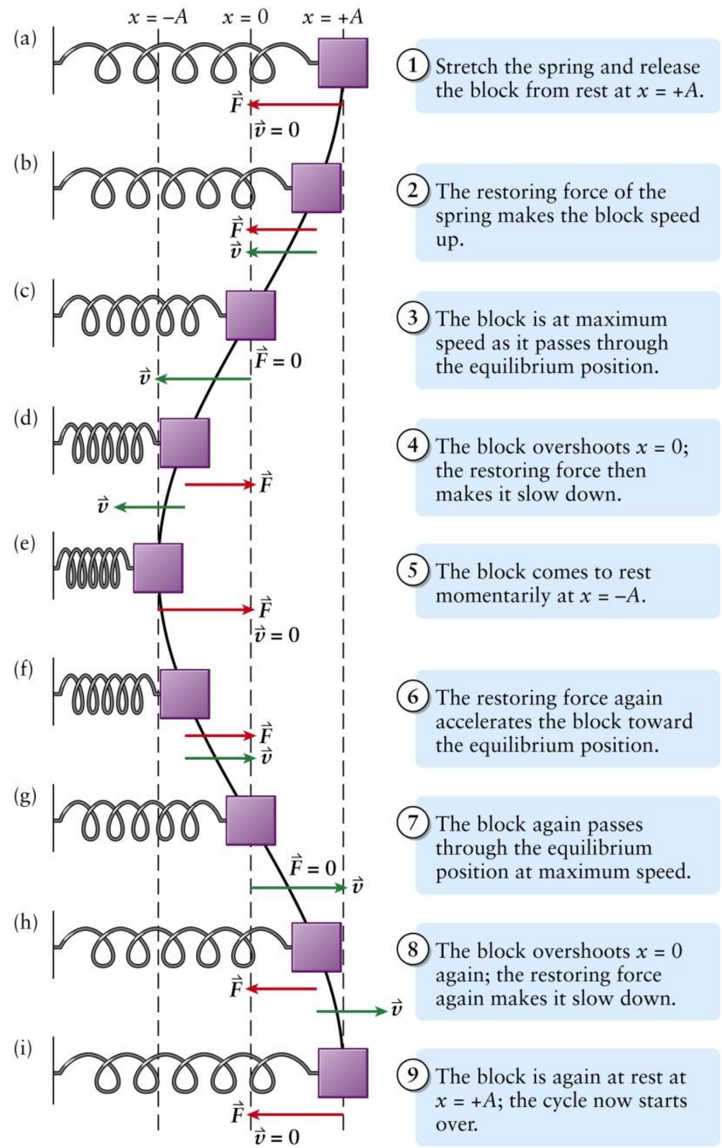


- fT=1 – freq f(Hz) time period T(s) =1
- **f=1/T   □ = 2π f   □T=2π   T =2π /□**

What causes periodic motion?

- If a body attached to a spring is displaced from its equilibrium position, the spring exerts a *restoring force* on it, which tends to restore the object to the equilibrium position. This force causes *oscillation* of the system, or *periodic motion*.
- Figure at the right illustrates the restoring force F_x.





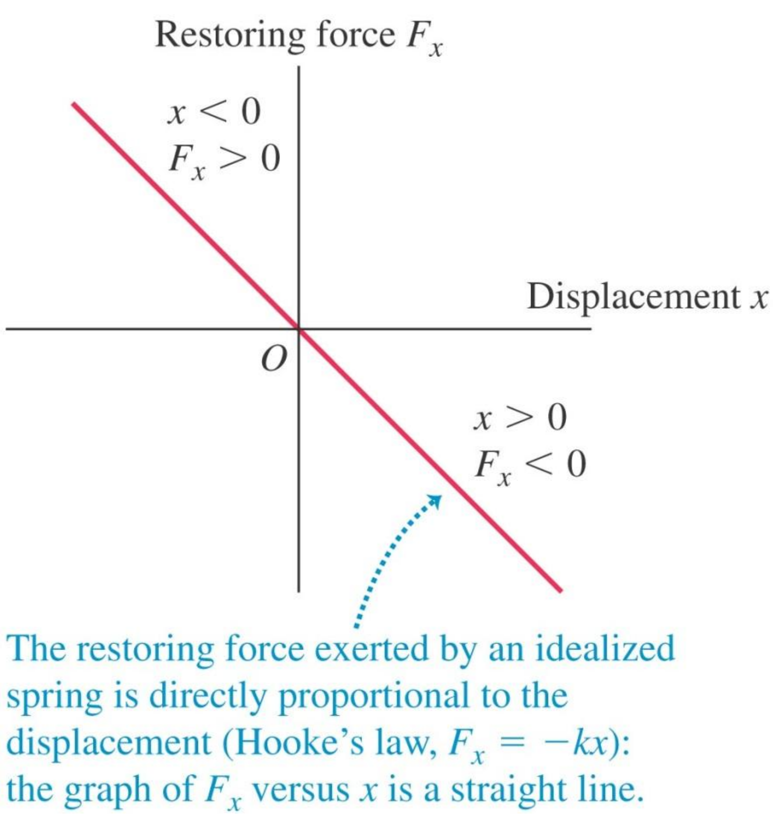
x = −A
x = 0
x = +A
(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)
(i)
F
v = 0
F = 0
v
1 Stretch the spring and release the block from rest at x = +A.
2 The restoring force of the spring makes the block speed up.
3 The block is at maximum speed as it passes through the equilibrium position.
4 The block overshoots x = 0; the restoring force then makes it slow down.
5 The block comes to rest momentarily at x = −A.
6 The restoring force again accelerates the block toward the equilibrium position.
7 The block again passes through the equilibrium position at maximum speed.
8 The block overshoots x = 0 again; the restoring force again makes it slow down.
9 The block is again at rest at x = +A; the cycle now starts over.

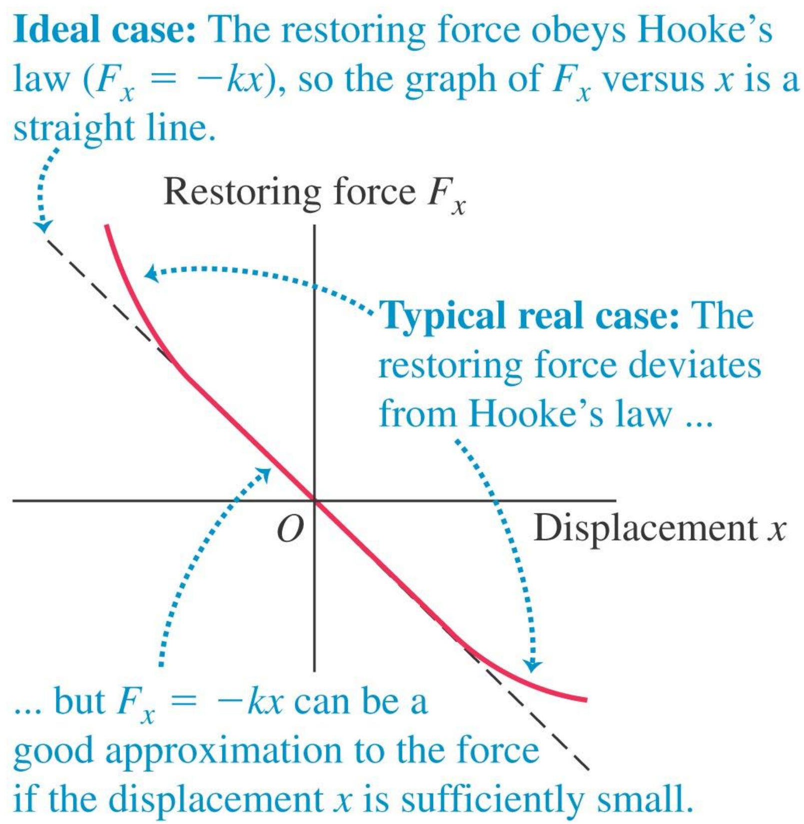
Characteristics of periodic motion

- The *amplitude*, A, is the maximum magnitude of displacement from equilibrium.
- The *period*, T, is the time for one cycle.
- The *frequency*, f, is the number of cycles per unit time.
- The *angular frequency*, ω, is 2π times the frequency: $\omega = 2\pi f$.
- The frequency and period are reciprocals of each other: $f = 1/T$ and $T = 1/f$.

Simple harmonic motion (SHM)
Simple Harmonic Oscillator (SHO)

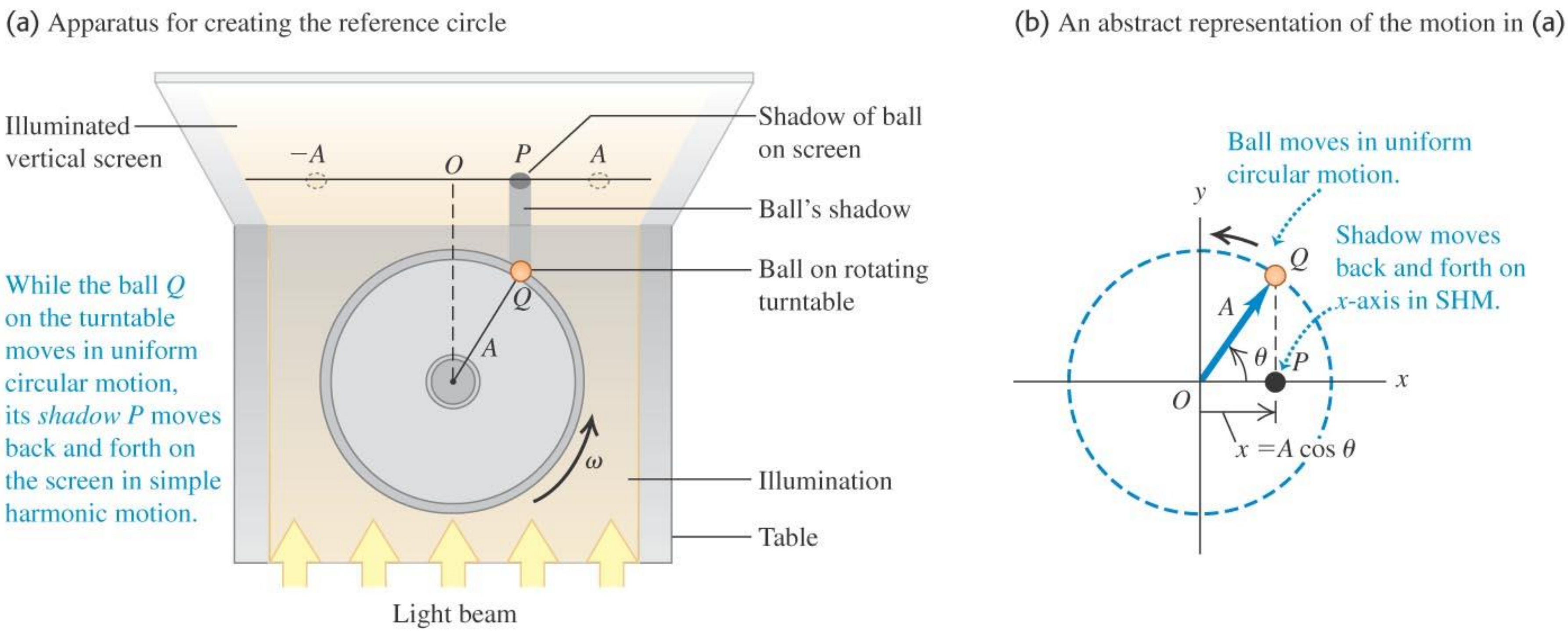
- When the restoring force is *directly proportional* to the displacement from equilibrium, the resulting motion is called *simple harmonic motion* (SHM).
- An ideal spring obeys Hooke's law, so the restoring force is $F_x = -kx$, which results in simple harmonic motion.





Simple harmonic motion viewed as a projection

- Simple harmonic motion is the projection of uniform circular motion onto a diameter

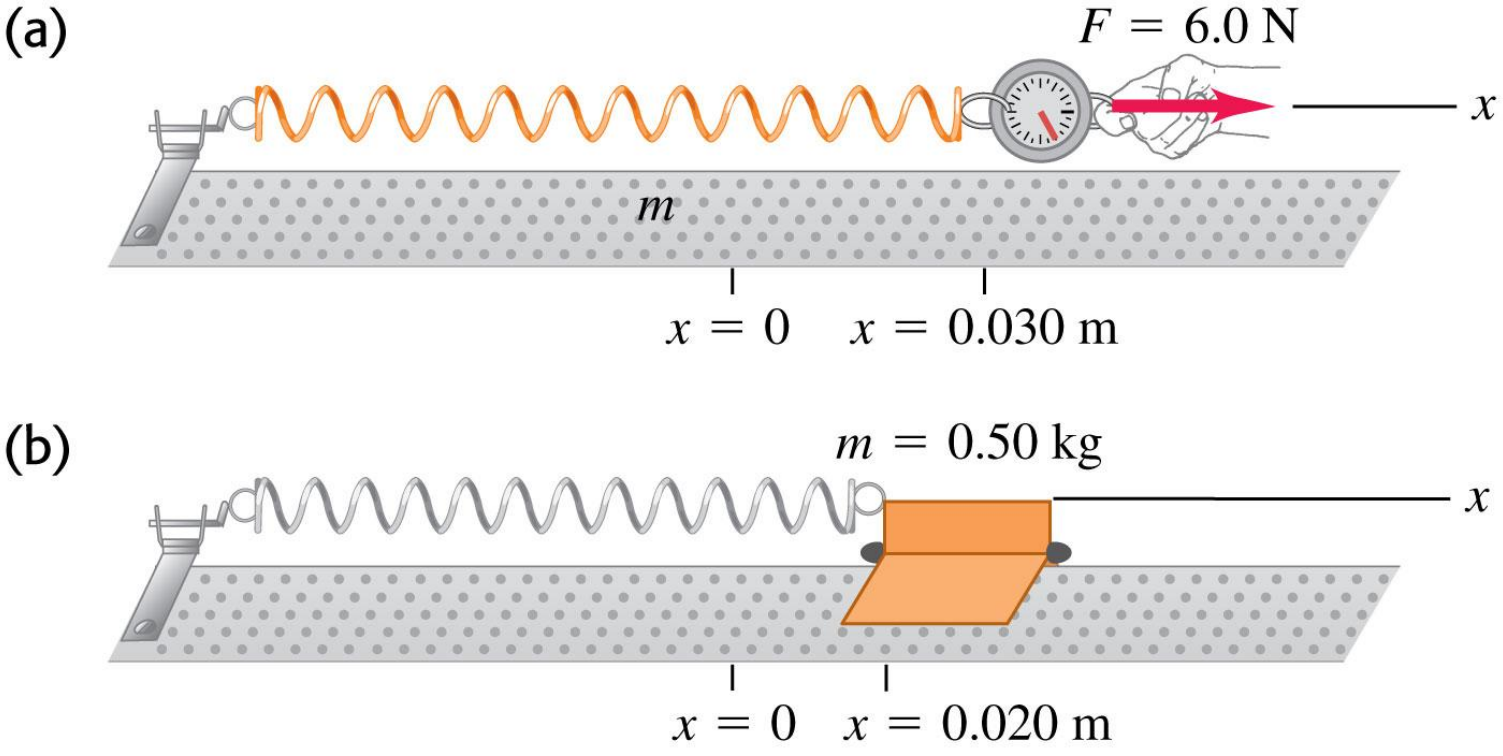


Characteristics of SHM

- For a body vibrating by an ideal spring:

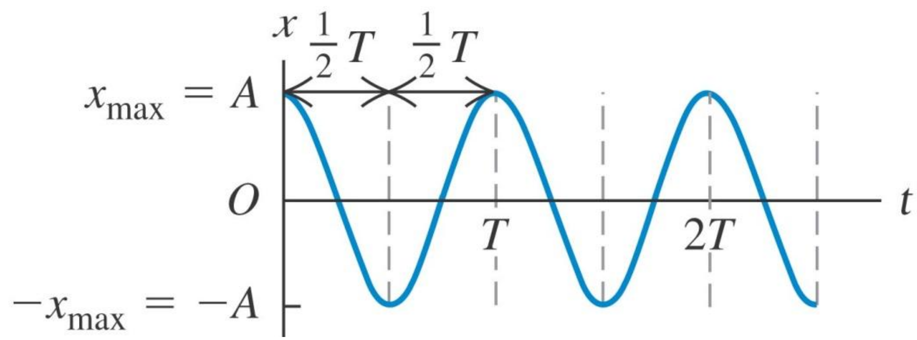
$$\omega = \sqrt{\frac{k}{m}} \quad f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

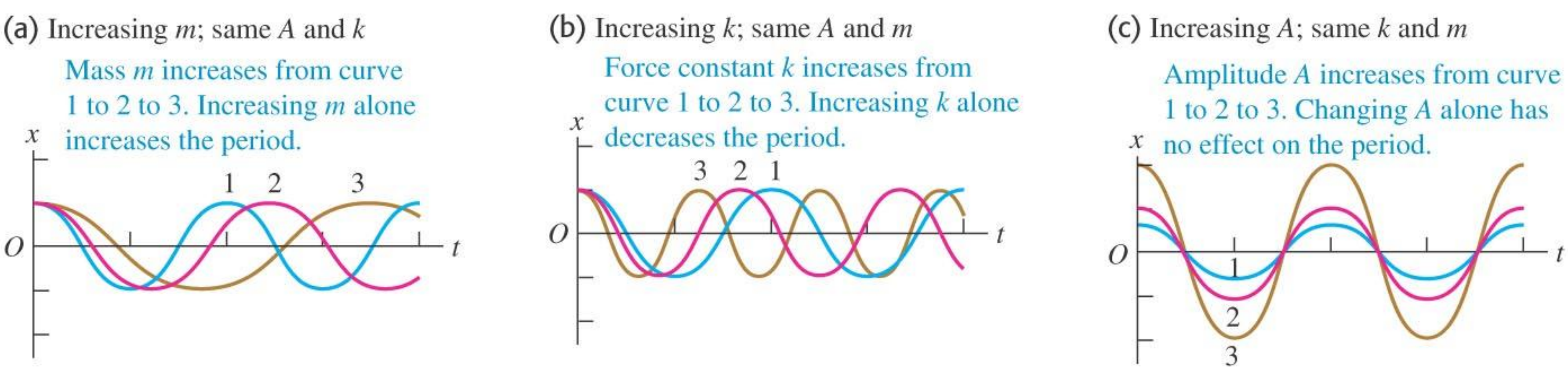
- Follow Example 14.2 and Figure 14.8 below.



Displacement as a function of time in SHM

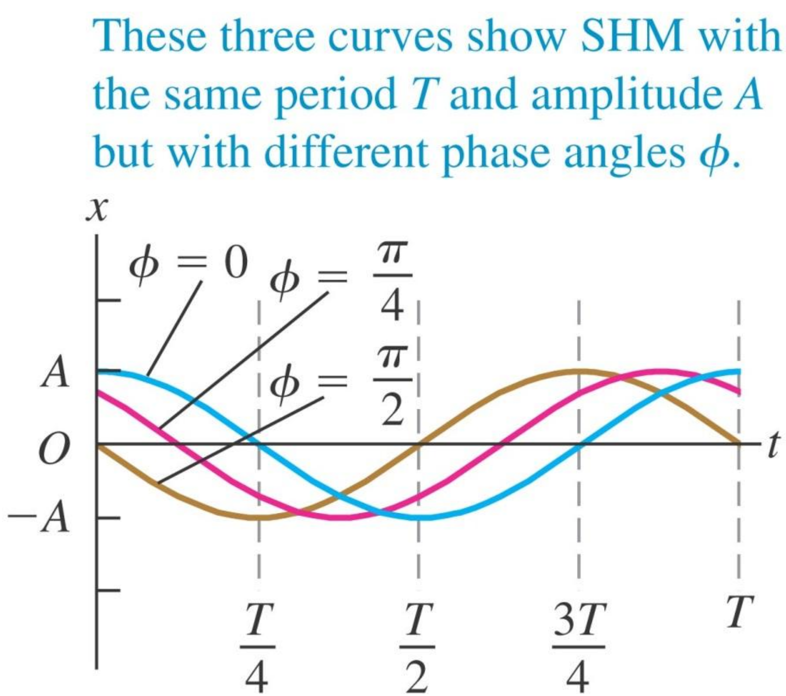
- The displacement as a function of time for SHM with phase angle ϕ is $x = A\cos(\omega t + \phi)$
- Changing m, A, or k changes the graph of x versus t, as shown below.



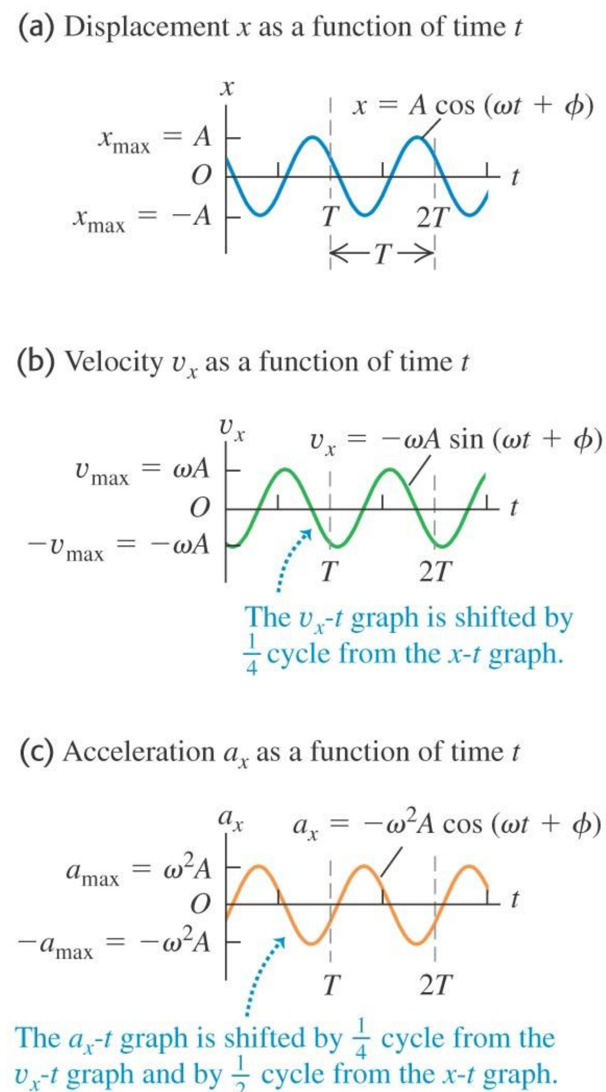


Displacement, velocity, and acceleration

- The graph below shows the effect of different phase angles.

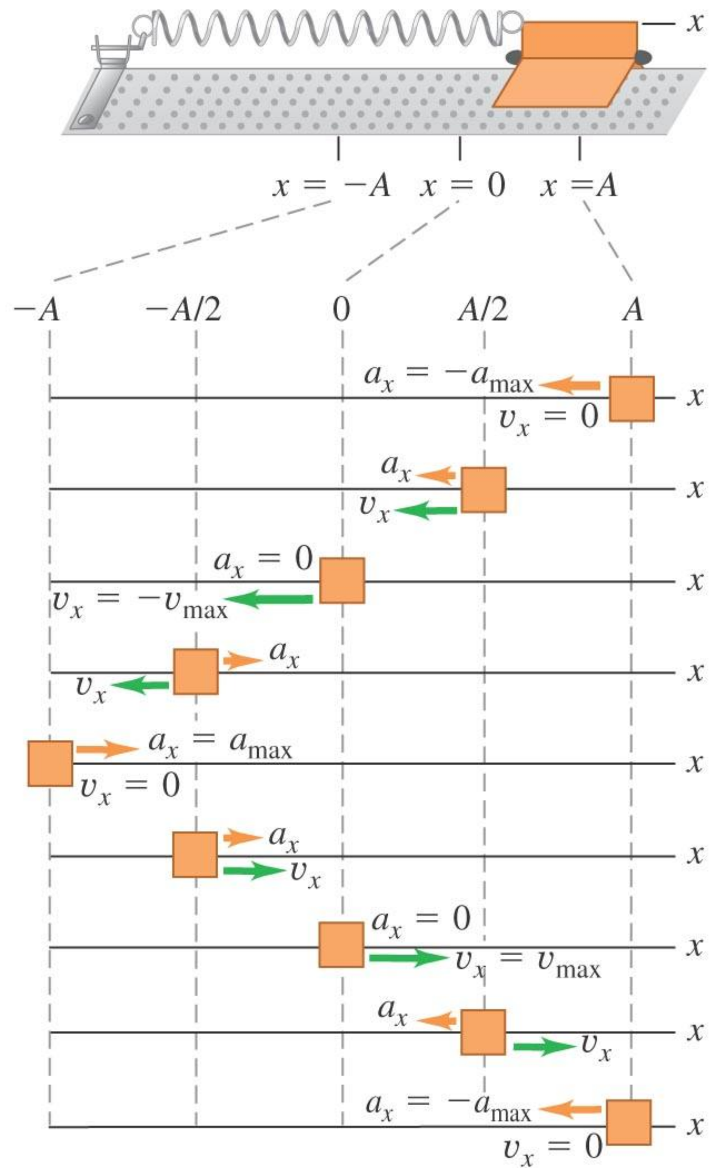


- The graphs below show x, v_x, and a_x for $\phi = \pi/3$.



Behavior of v_x and a_x during one cycle

- Figure shows how v_x and a_x vary during one cycle.



SHO - mass and amplitude

An object on the end of a spring is oscillating in simple harmonic motion. If the amplitude of oscillation is doubled, how does this affect the oscillation period T and the object's maximum speed v_{max}?

A. T and v_{max} both double.

B. T remains the same and v_{max} doubles.

C. T and v_{max} both remain the same.

D. T doubles and v_{max} remains the same.

E. T remains the same and v_{max} increases by a factor of $\sqrt{2}$.

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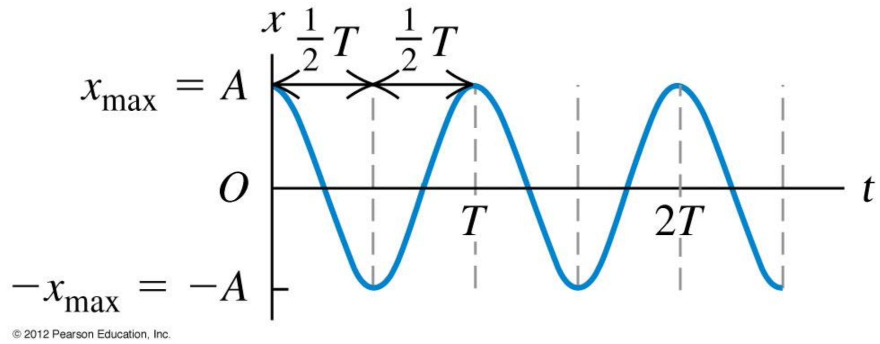
This is an *x*-*t* graph for an object in simple harmonic motion.



At which of the following times does the object have the *most negative velocity* v_x?

A. $t = T/4$

B. $t = T/2$

C. $t = 3T/4$

D. $t = T$

E. Two of the above are tied for most negative velocity

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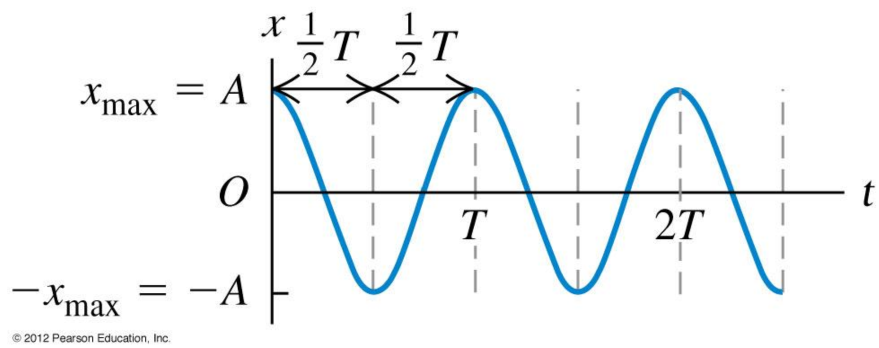

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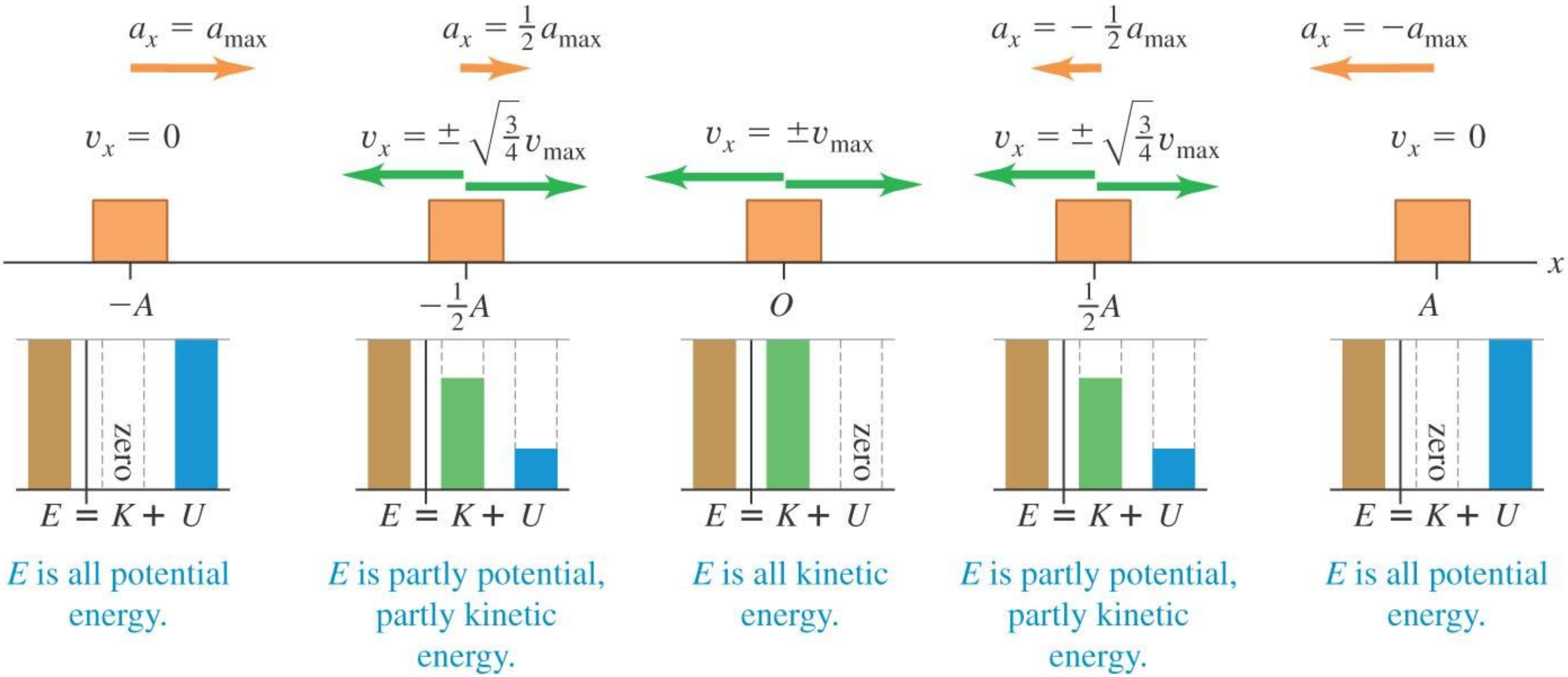
Energy in SHM

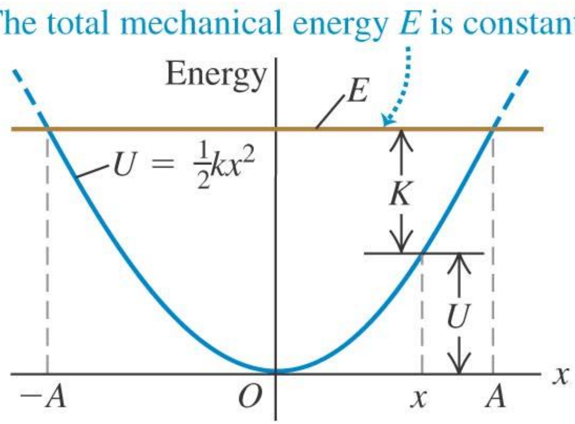
- The total mechanical energy $E = K + U$ is conserved in SHM:

$$E = 1/2\ mv_x^2 + 1/2\ kx^2 = 1/2\ kA^2 = 1/2\ mv_{x\text{-}maximum}^2 = \text{constant}$$

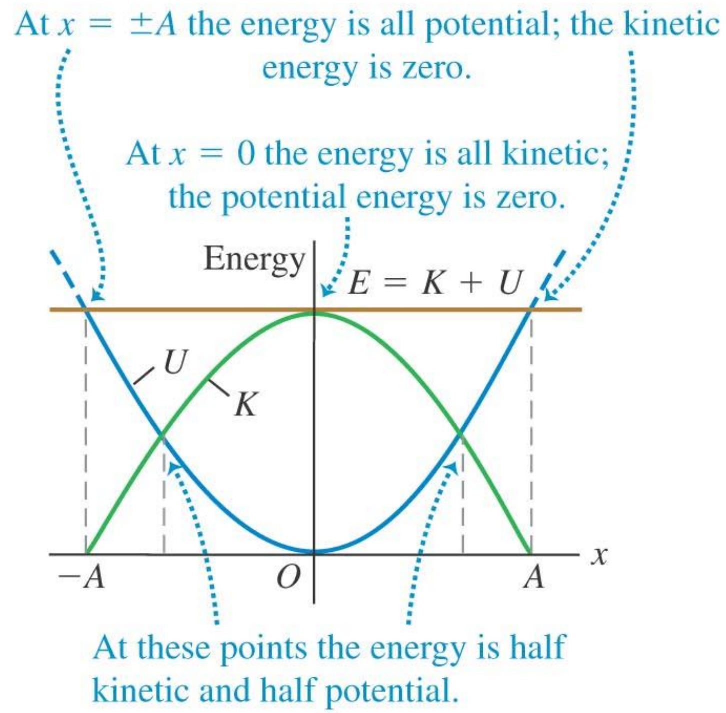


Energy diagrams for SHM

(a) The potential energy U and total mechanical energy E for a body in SHM as a function of displacement x



(b) The same graph as in (a), showing kinetic energy K as well

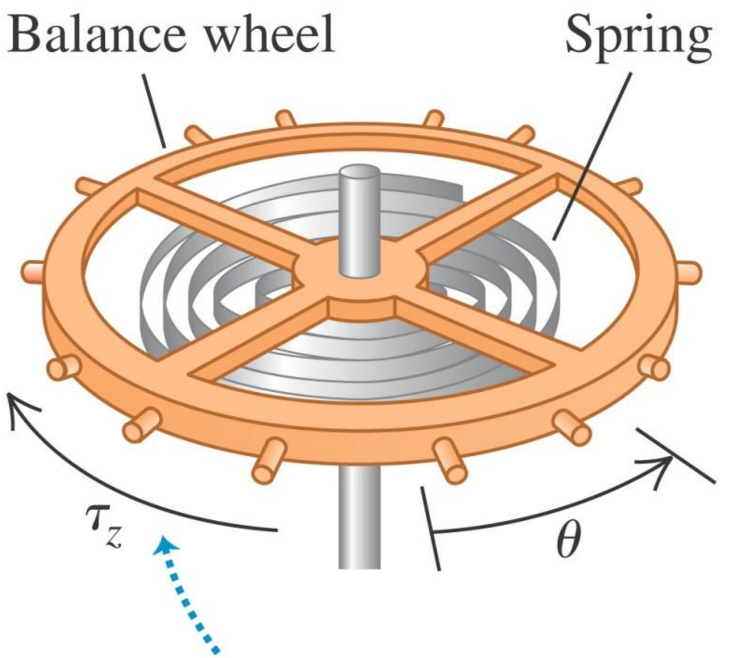


Vertical SHM – Mass and Spring Gravity does NOT matter here

- If a body oscillates vertically from a spring, the restoring force has magnitude kx. Therefore the vertical motion is SHM.
- For a pendulum Gravity DOES matter.

Angular SHM – old mechanical watch

- A coil spring exerts a restoring torque $\tau_z = -\kappa\theta$, where κ is called the *torsion constant* of the spring.
- The result is *angular* simple harmonic motion.

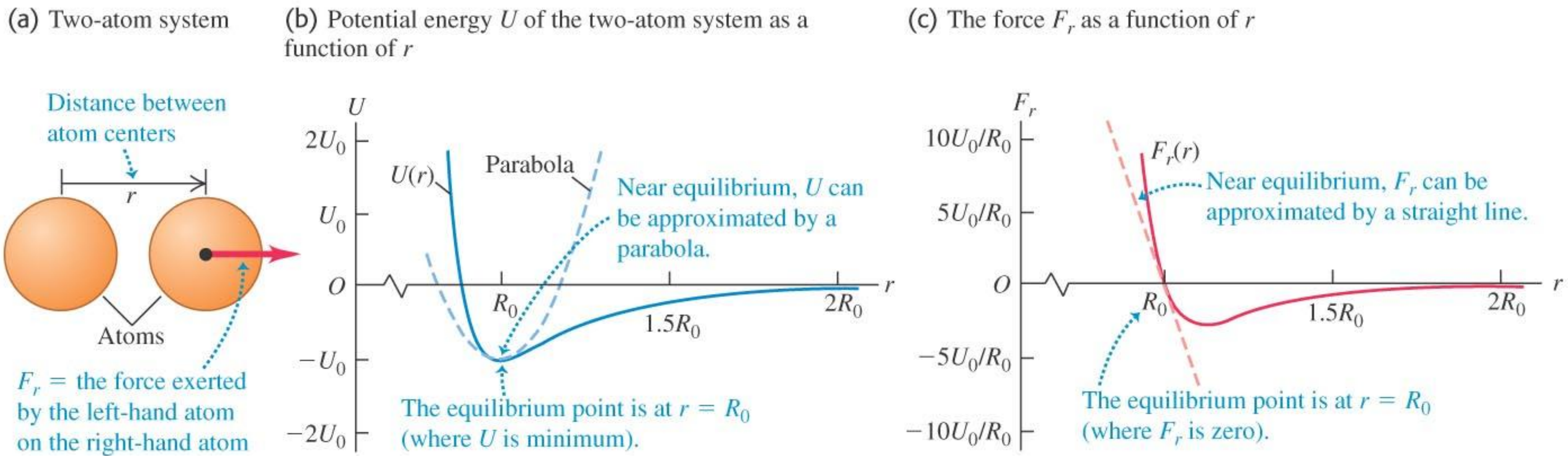


The spring torque τ_z opposes the angular displacement θ.

Vibrations of molecules

Intermolular forces

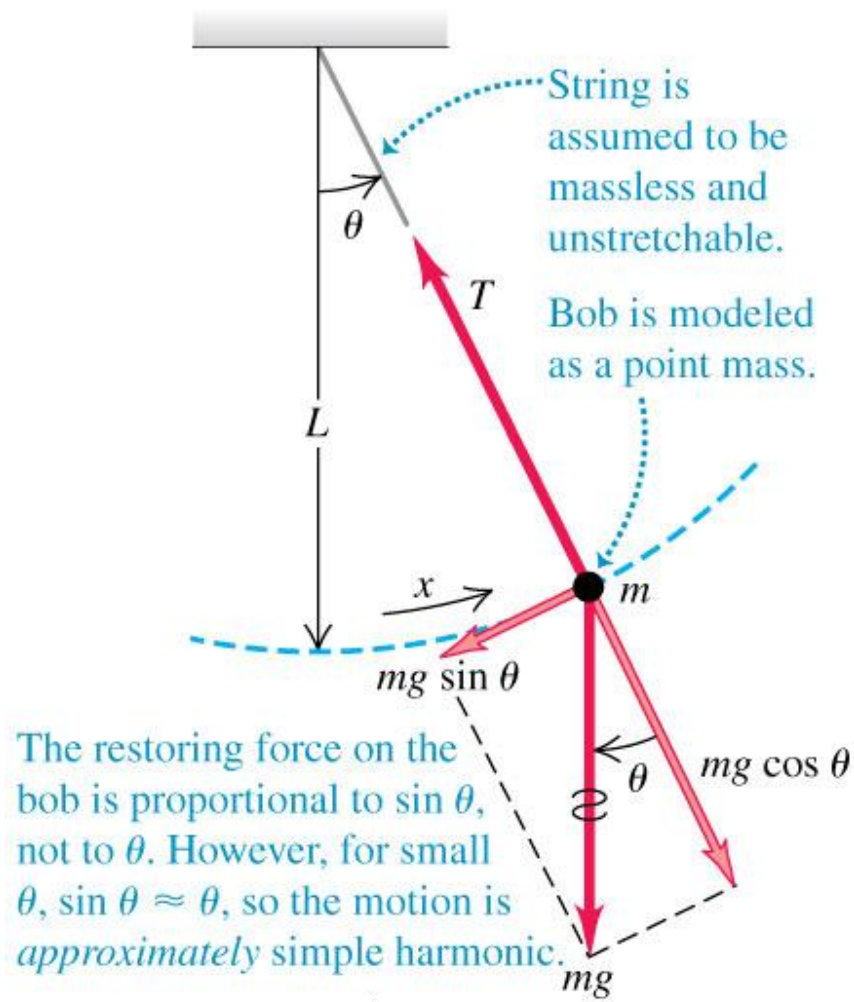
- Figure shows two atoms having centers a distance r apart, with the equilibrium point at $r = R_0$.
- If they are displaced a small distance x from equilibrium, the restoring force is $F_r = -(72U_0/R_0^2)x$, so $k = 72U_0/R_0^2$ and the motion is SHM.
- Van der Waal like forces.



The simple pendulum

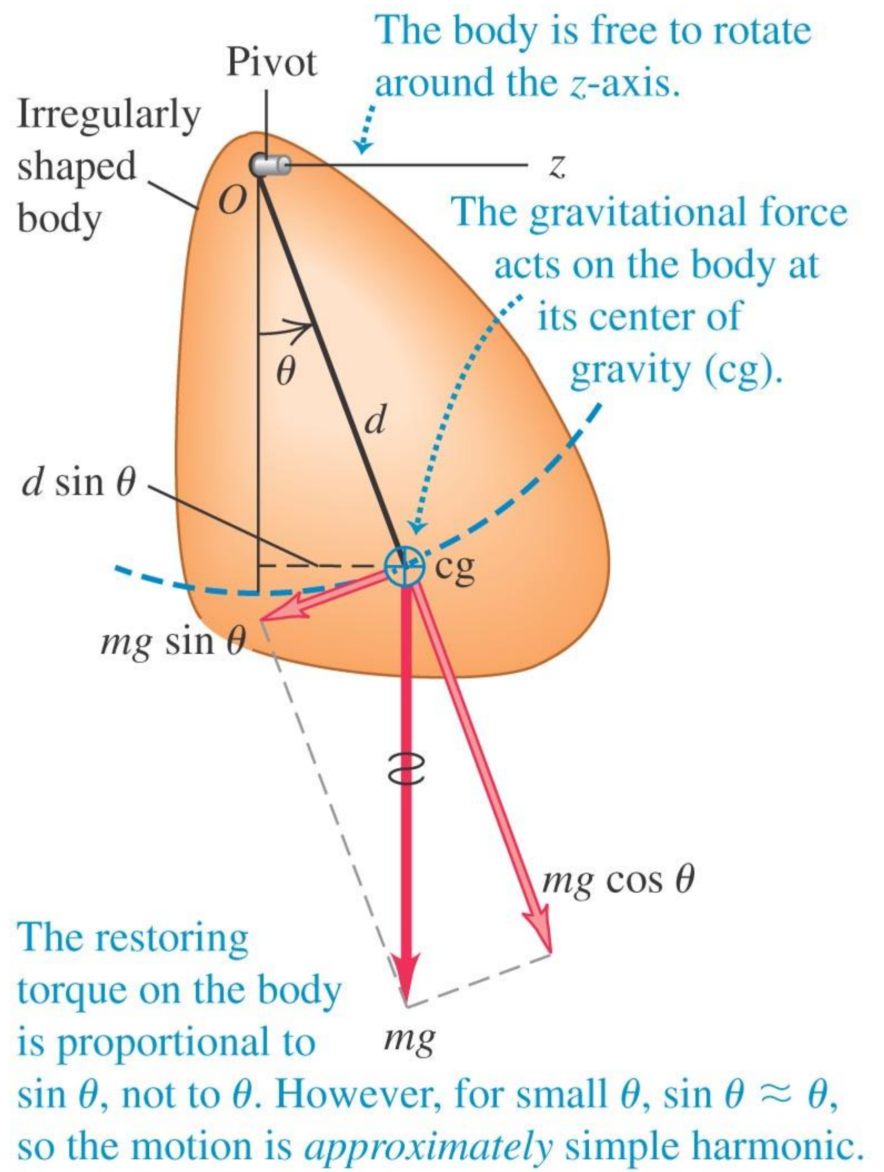
- A *simple pendulum* consists of a point mass (the bob) suspended by a massless, unstretchable string.
- If the pendulum swings with a small amplitude θ with the vertical, its motion is simple harmonic.
- I□=□ , I = moment inertia = mL^2
- □ = torque = L*m*g sin(□)
- □ = angular accel = d^2 □/dt^2
- Eq. motion d^2 □/dt^2 = (g/L) sin(□) ~ (g/L) □
- Solution is □(t) = Asin(□t+□) - SHO
- A – amp, □ - phase – both set by initial cond
- □ = $(g/L)^{1/2}$ angular freq (rad/s)
- T=2π/□ = 2π $(L/g)^{1/2}$
- Note T ~ $L^{1/2}$ and $g^{-1/2}$

(b) An idealized simple pendulum



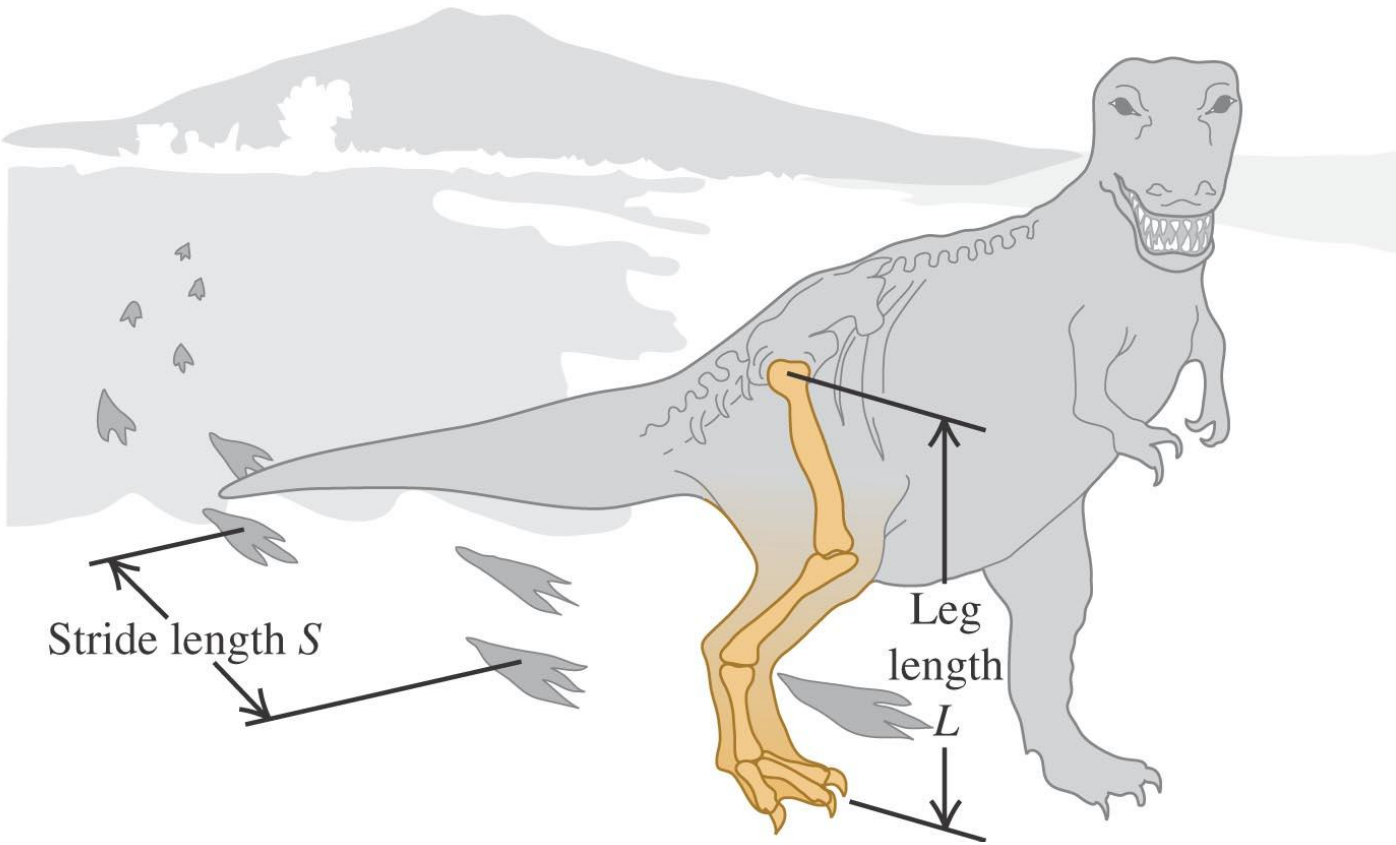
The physical pendulum

- A *physical pendulum* is any real pendulum that uses an extended body instead of a point-mass bob.
- For small amplitudes, its motion is simple harmonic.
- Same solution as simple pendulum – ie SHO.
- □ = (g/L)$^{1/2}$ angular freq (rad/s)
- T=2π/□ = 2π (L/g)$^{1/2}$



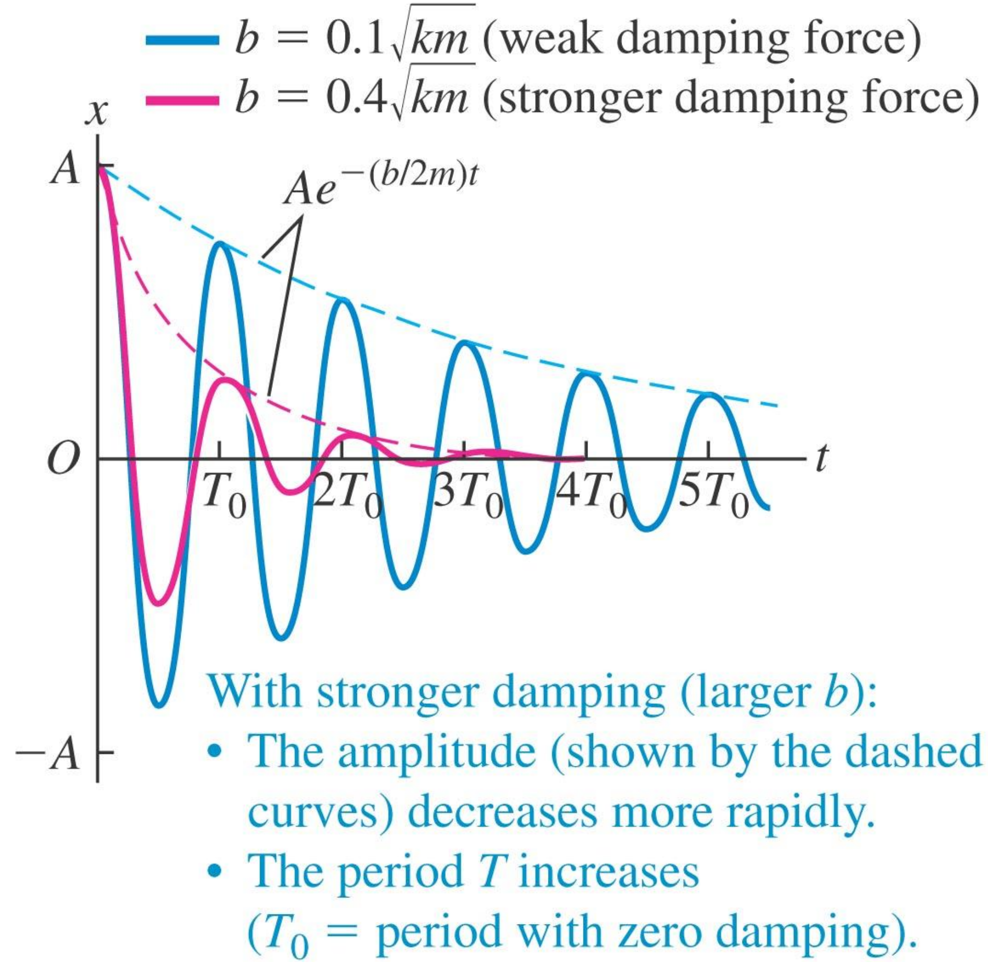
Tyrannosaurus rex and the physical pendulum

- We can model the leg of *Tyrannosaurus rex* as a physical pendulum.
- Unhappy T Rex – cannot use social media in class.



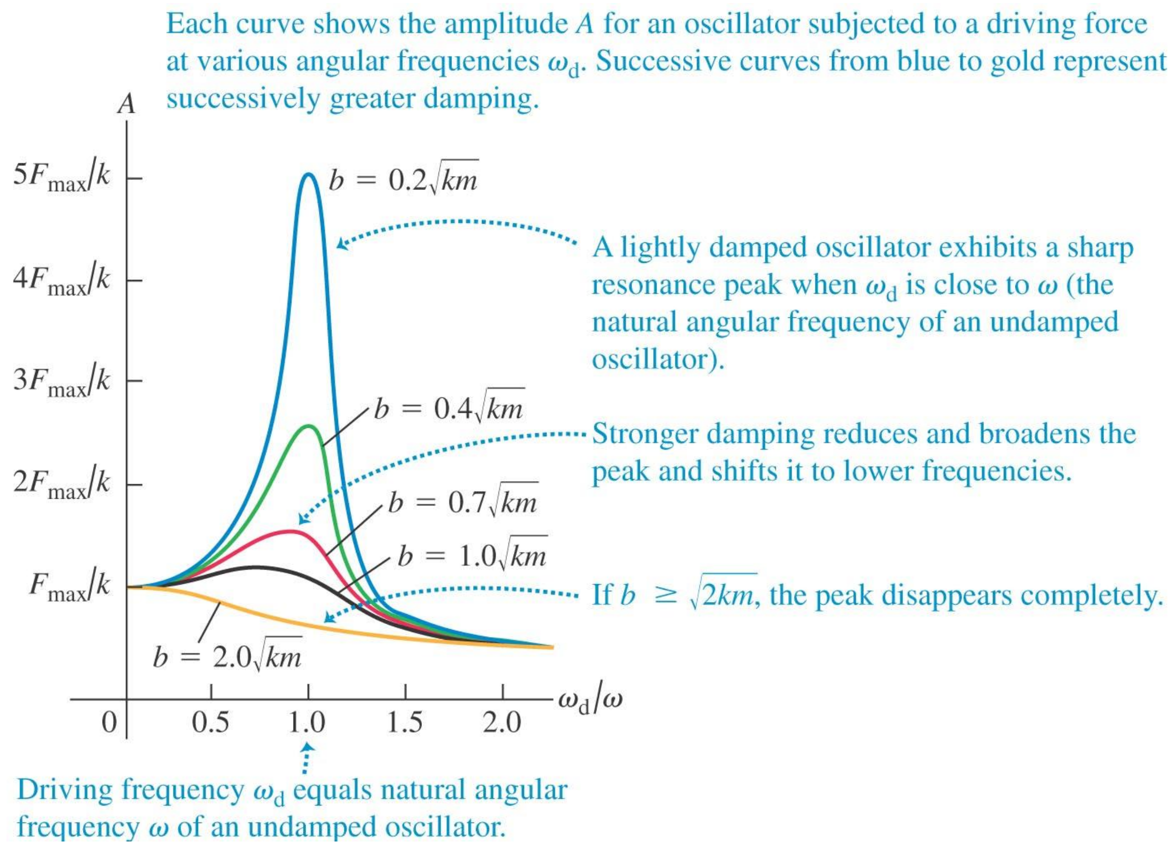
Damped oscillations

- Real-world systems have some dissipative forces that decrease the amplitude.
- The decrease in amplitude is called *damping* and the motion is called *damped oscillation*.
- Figure illustrates an oscillator with a small amount of damping.
- The mechanical energy of a damped oscillator decreases continuously.



Forced oscillations and resonance

- A *forced oscillation* occurs if a *driving force* acts on an oscillator.
- *Resonance* occurs if the frequency of the driving force is near the *natural frequency* of the system.



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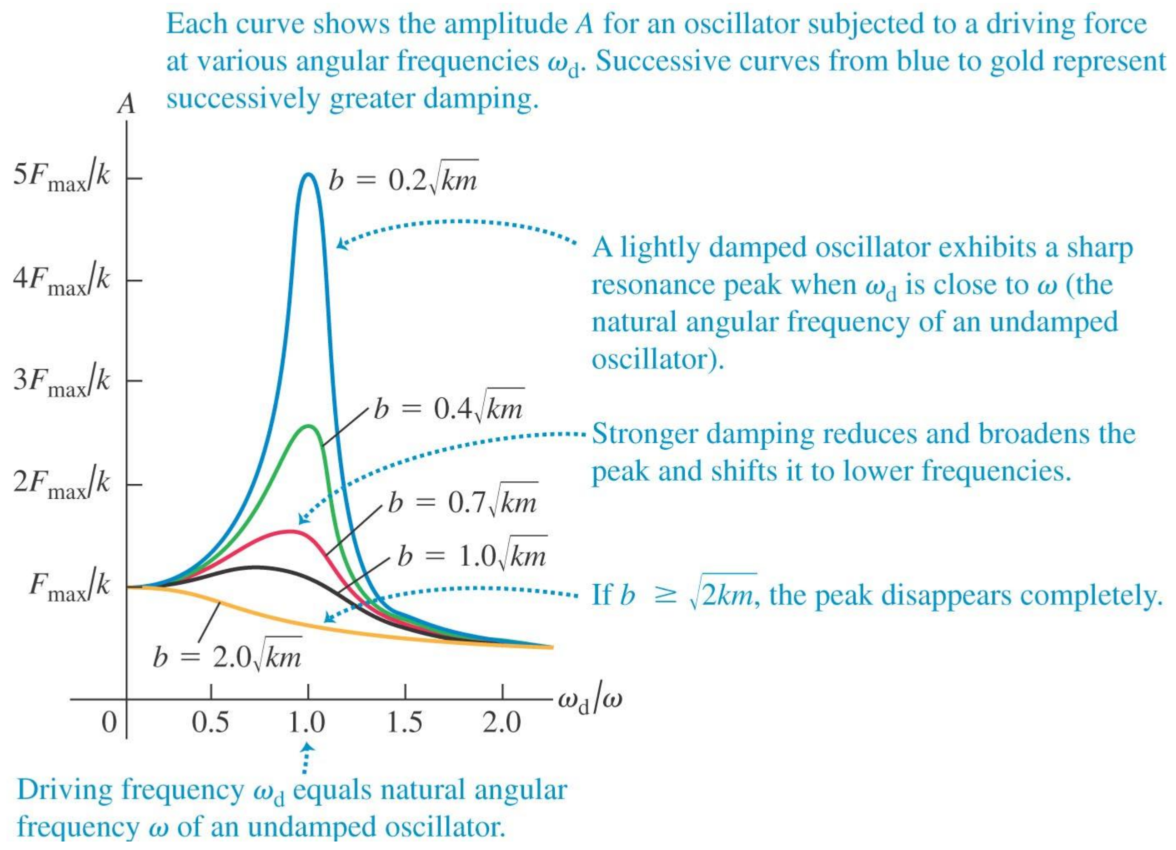
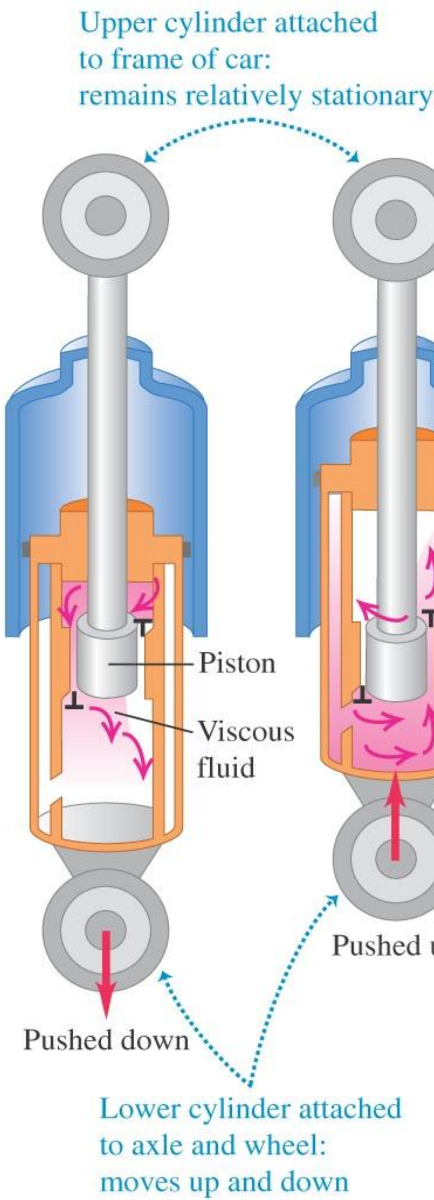


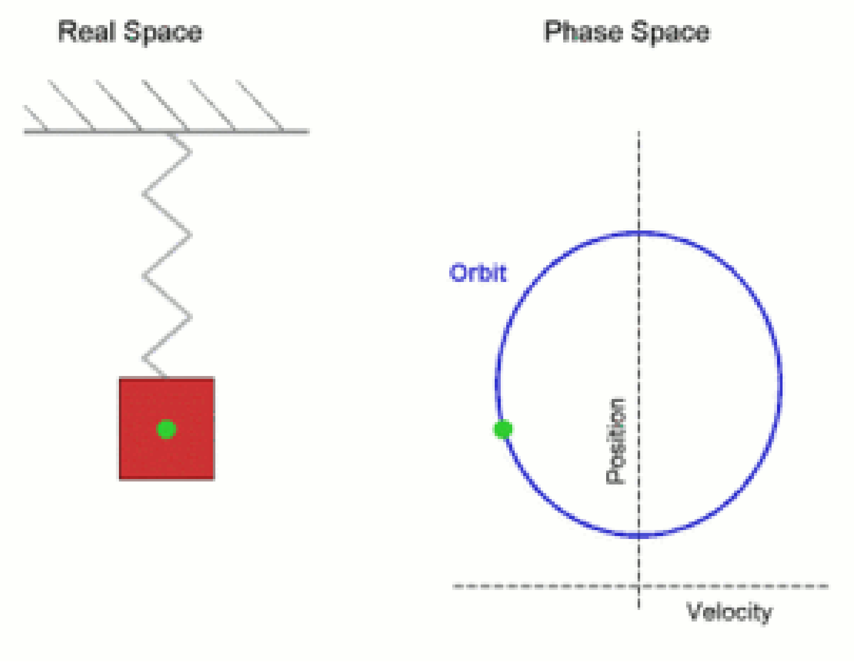
Car shock absorbers - Damped oscillations



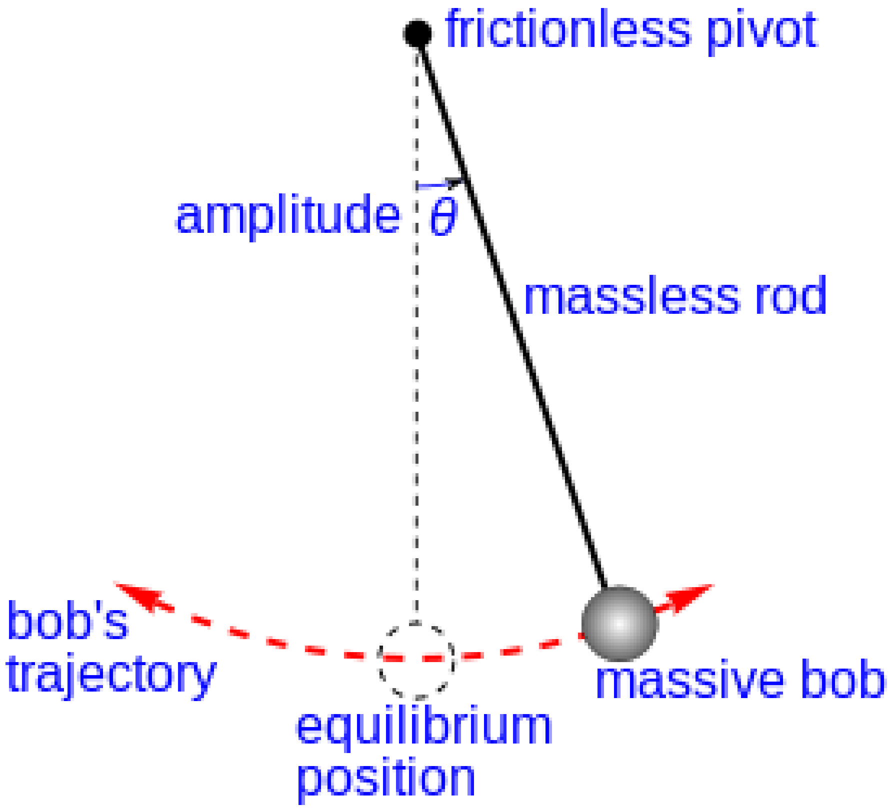
Forced oscillations and resonance
Structural Failure

- Nov 7, 1940
- The Tacoma Narrows Bridge suffered spectacular structural failure
- Wind driven osc - too much resonant energy. Too little damping
- https://www.youtube.com/watch?v=nFzu6CNtqec

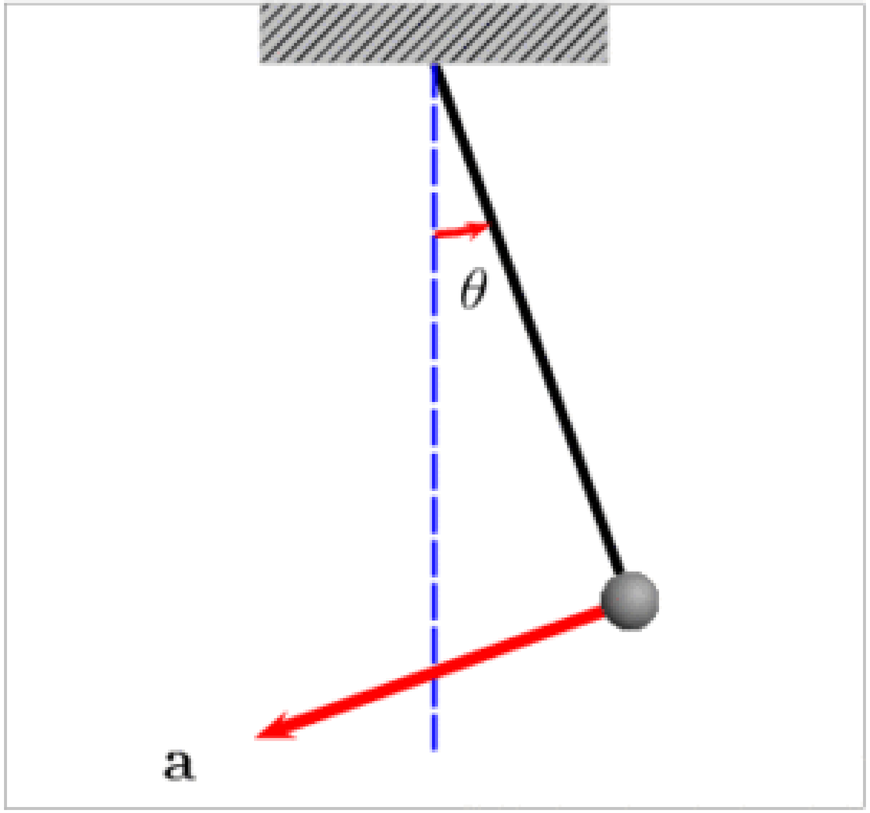
Simple Harmonic Oscillator (SHO)



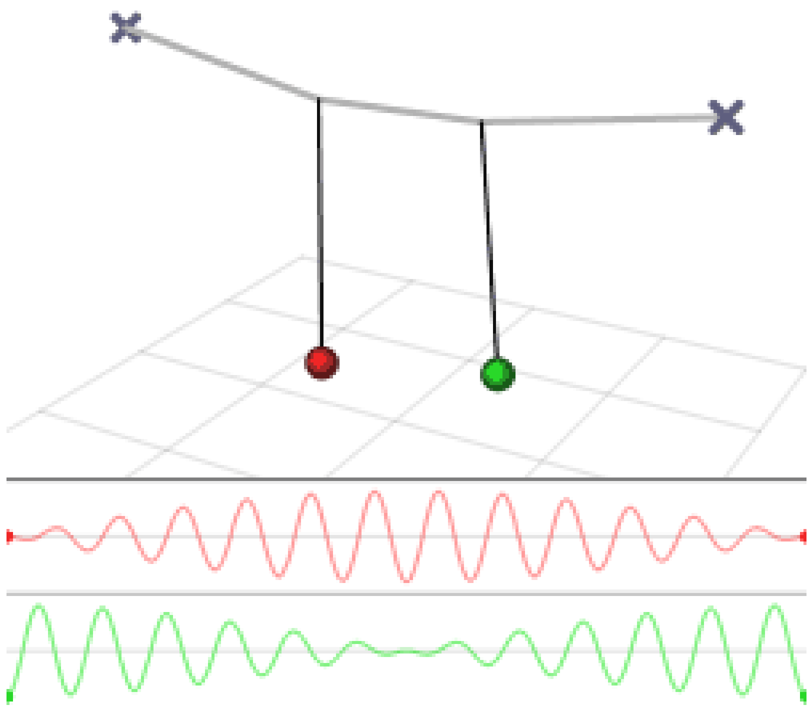
Pendulum


frictionless pivot
amplitude
θ
massless rod
bob's
trajectory
equilibrium
position
massive bob

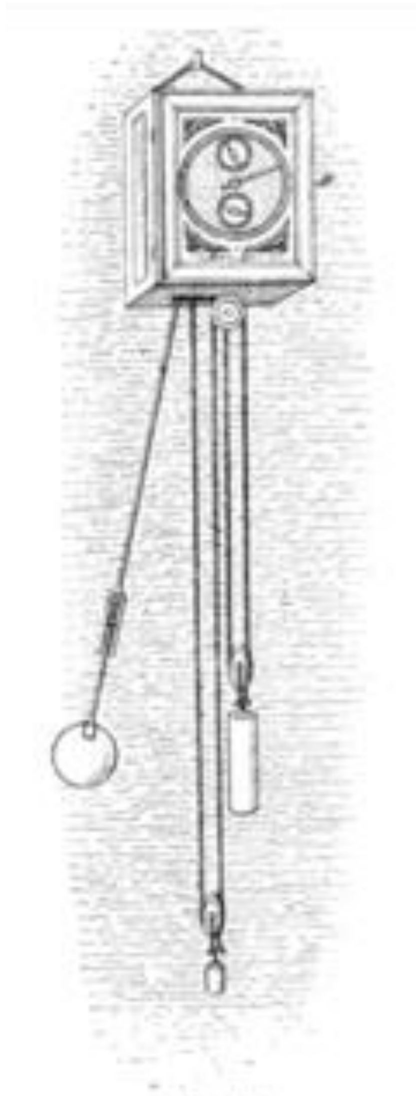
Simple Pendulum

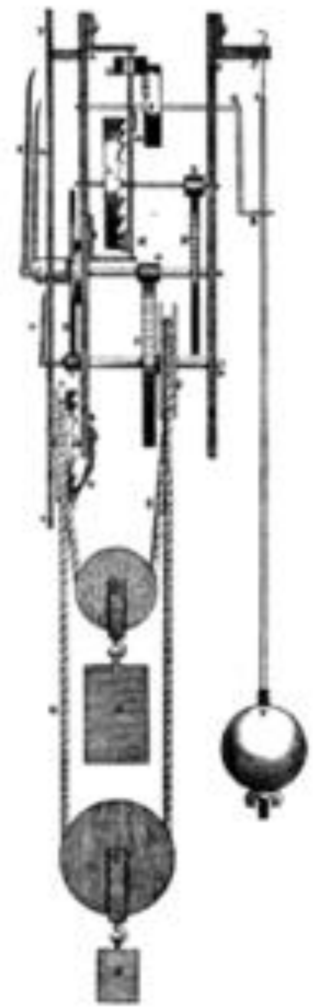


Two pendulums – same natural freq
Coupled on wire



Christian Huygens First Pendulum Clock 1656





US Time Standard 1909 to 1929

Pendulum is in low pressure vessel

NBS – National Bureau of Standards – now NIST (Natl Inst Sci and Tech)

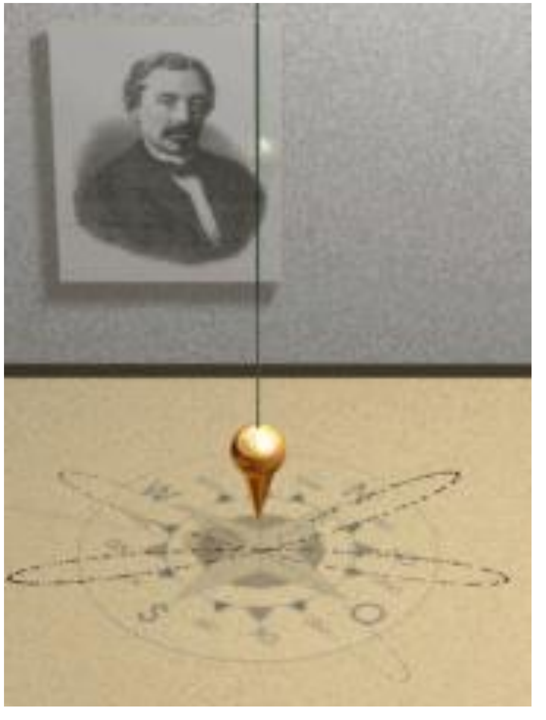
Riefler regulator

Vacuum Pendulum – 1 sec / year!!

Synchronized to second pendulum clock

Foucault Pendulum 1851

Precession of Pendulum
Showed Earth Rotates



Seconds Pendulum – 2 sec period Used to Measure Gravity