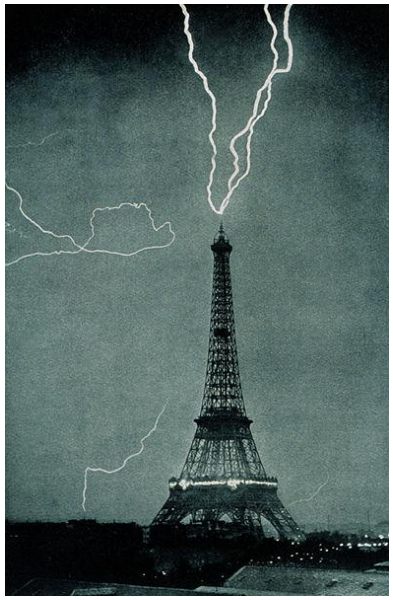
Chapter 16

Electrostatics I

Electrostatics – NOT Really Electrodynamics



Electric Charge – Some history

- Historically people knew of electrostatic effects
- Hair attracted to amber rubbed on clothes
- People could generate "sparks"
- Recorded in ancient Greek history
- 600 BC Thales of Miletus notes effects
- 1600 AD - William Gilbert coins Latin term *electricus* from Greek *ηλεκτρον* (*elektron*) – Greek term for Amber
- 1660 Otto von Guericke – builds electrostatic generator
- 1675 Robert Boyle – show charge effects work in vacuum
- 1729 Stephen Gray – discusses insulators and conductors
- 1730 C. F. du Fay – proposes two types of charges – can cancel
- Glass rubbed with silk – glass charged with *"vitreous electricity"*
- Amber rubbed with fur – Amber charged with *"resinous electricity"*

A little more history

- 1750 Ben Franklin proposes “vitreous” and “resinous” electricity are the same ‘electricity fluid” under different “pressures”
- He labels them “positive” and “negative” electricity
- Proposaes “conservation of charge”
- June 15 1752(?) Franklin flies kite and “collects” electricity
- 1839 Michael Faraday proposes “electricity” is all from two opposite types of “charges”
- We call “positive” the charge left on glass rubbed with silk
- Today we would say ‘electrons” are rubbed off the glass

Torsion Balance

- Charles-Augustin de Coulomb - 1777

Used to measure force from electric charges and to measure force from gravity

$\tau = -\kappa\theta$ - **"Hooks law" for fibers**

(recall F = -kx for springs)

General Equation with damping

θ – angle

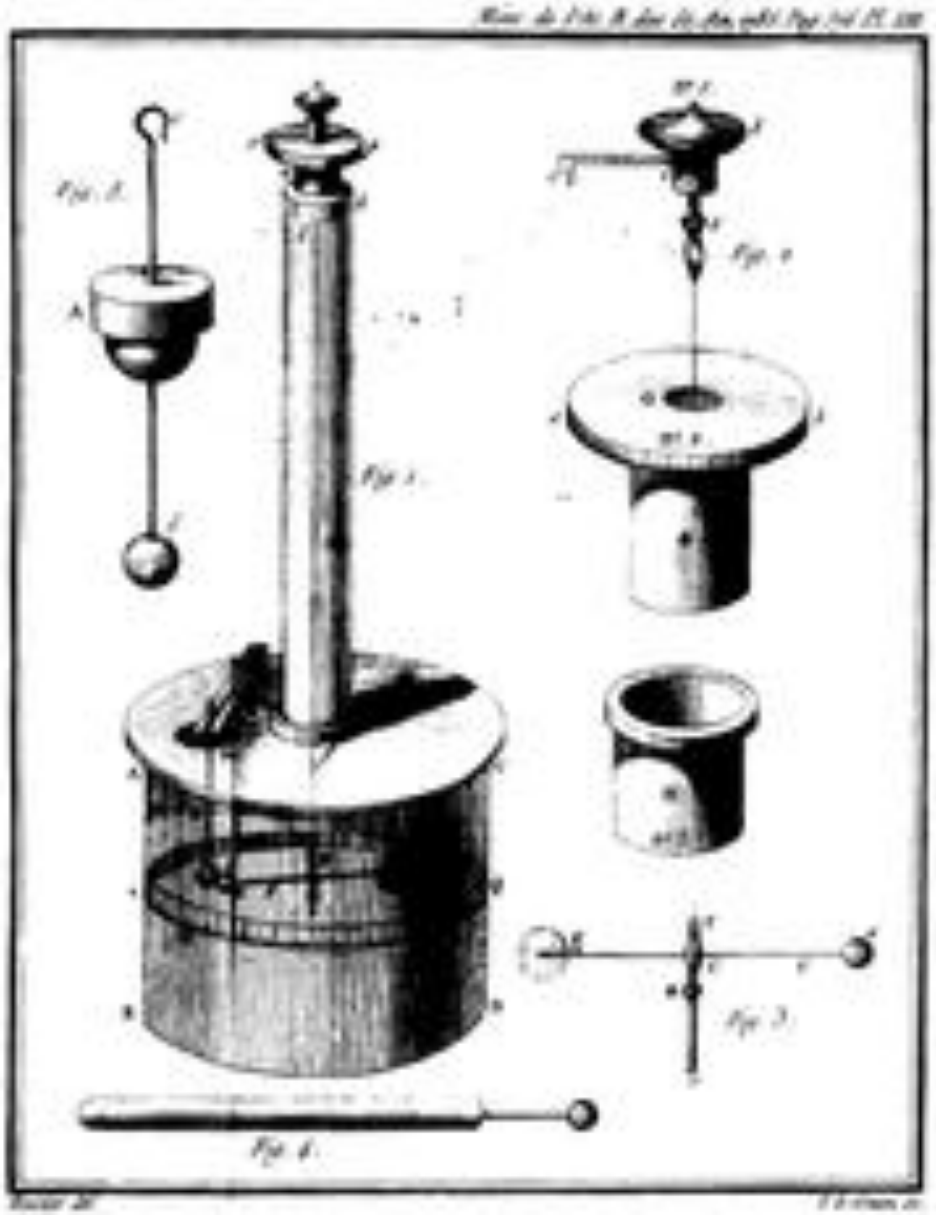
I – moment of inertia

C – damping coefficient

κ – torsion constant

τ – driving torque

$$I\frac{d^2\theta}{dt^2} + C\frac{d\theta}{dt} + \kappa\theta = \tau(t)$$



Solutions to the damped torsion balance

$$\theta = Ae^{-\alpha t}\cos\left(\omega t + \phi\right)$$

General solutions are damped oscillating terms – ie damped SHO
A = amplitude
t = time
α = damping frequency = 1/damping time (e folding time)
ϕ = phase shift
ω = resonant angular frequency

$$\alpha = C/2I$$

If we assume a lightly damped system where:

$$C \ll \sqrt{\kappa I}$$

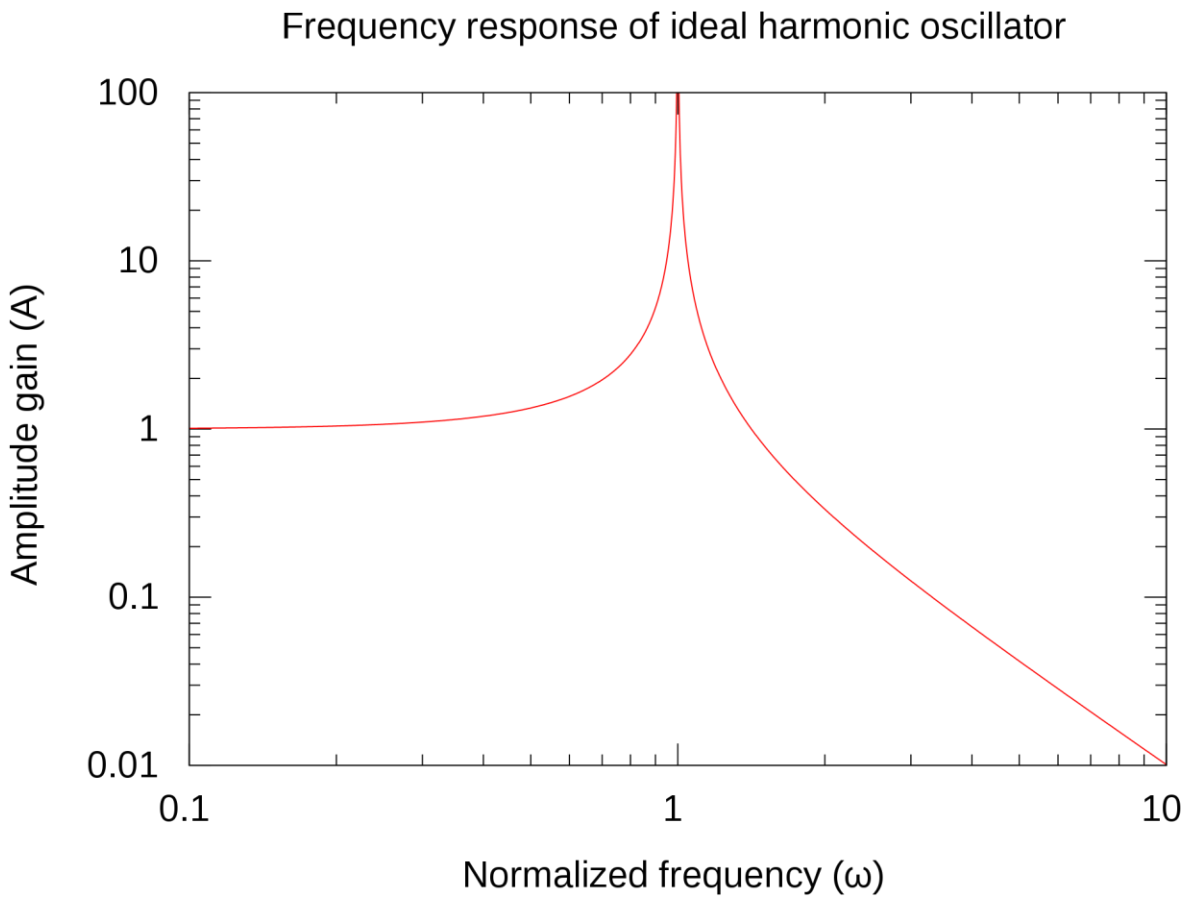
Then the resonant frequency is just the undamped resonant frequency

$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\kappa/I}$$

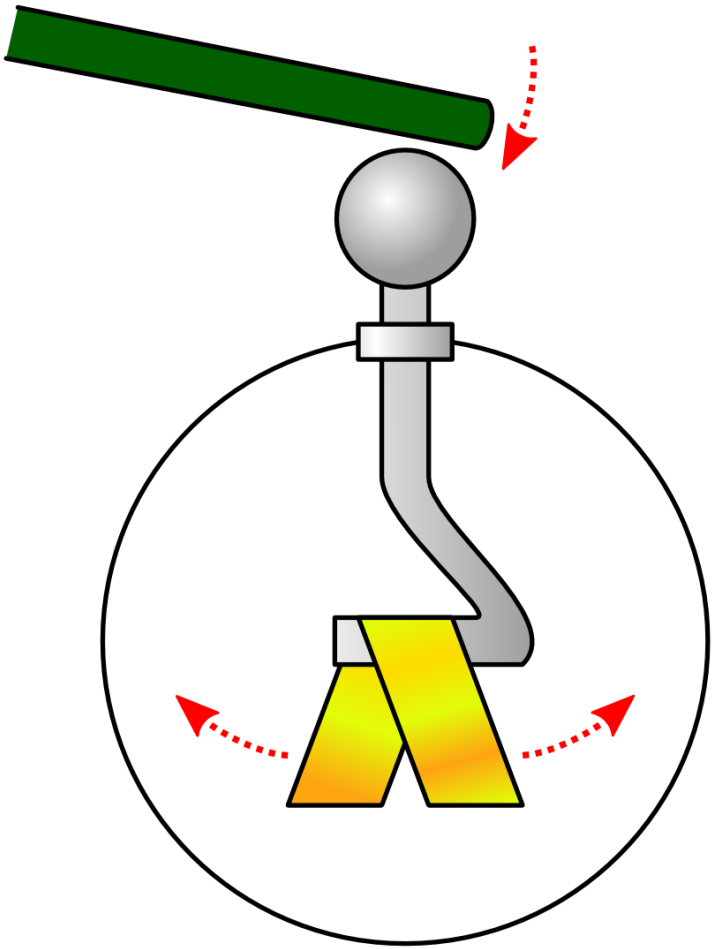
$\omega_n = \sqrt{(\kappa/I)}$ (ω_n = "natural undamped resonant freq")
recall for a spring with mass m that
$\omega = \sqrt{(k/m)}$ where k=spring constant

Amplitude vs freq – Bode Plot

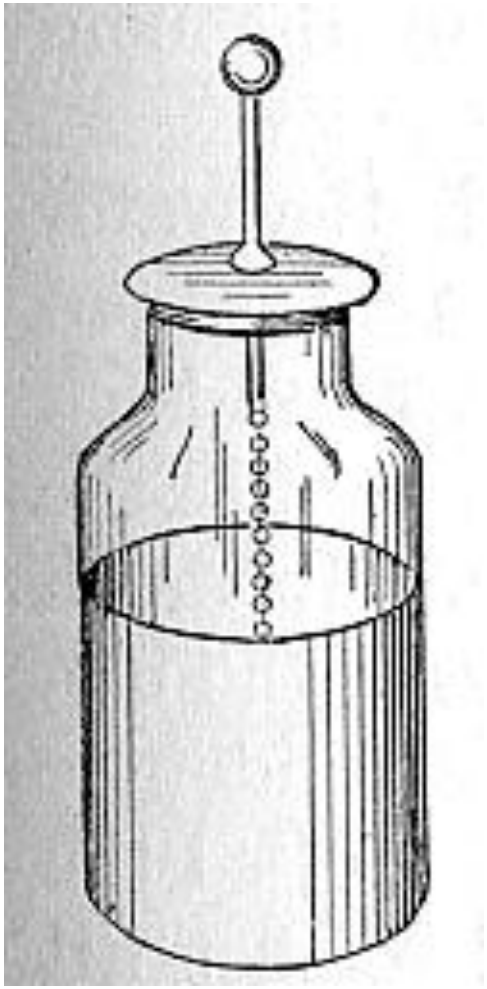
$$A = A(\zeta, \omega) = \frac{1}{\sqrt{(1-\omega^2)^2 + (2\zeta\omega)^2}}.$$

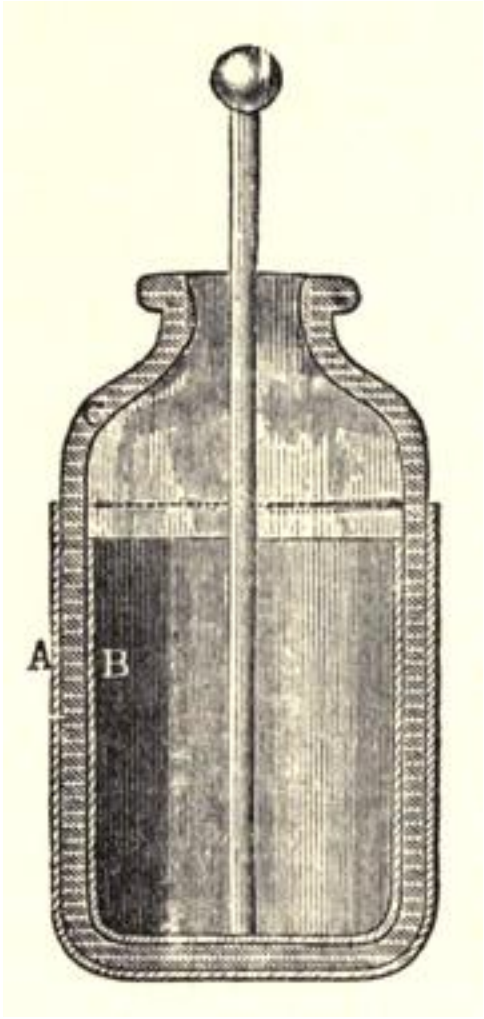


Gold leaf electroscope – used to show presence of charge
Gold leaf for gilding is about 100 nm thick!!



Leyden Jar – historical capacitor





Force between charges as measured on the lab with a torsion balance

$$F_C = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

ε_0 ~ 8.854 187 817 … x 10^{-12} Vacuum permittivity

$$\varepsilon_0 = \frac{1}{\mu_0 c_0^2}$$

μ_0 = Vacuum permeability (magnetic)
=$4\pi \times 10^{-7}$ H m^{-1} – defined exactly
c_0 = speed of light in vacuum

Coulombs “Law”

Define the electric field E = F/q where F is the force on a charge q
In the lab we measure an inverse square force law like gravity

For a point charge Q the E field at a distance r is given by Coulomb’s Law. It is a radial field and points away from a positive charge and inward towards a negative charge

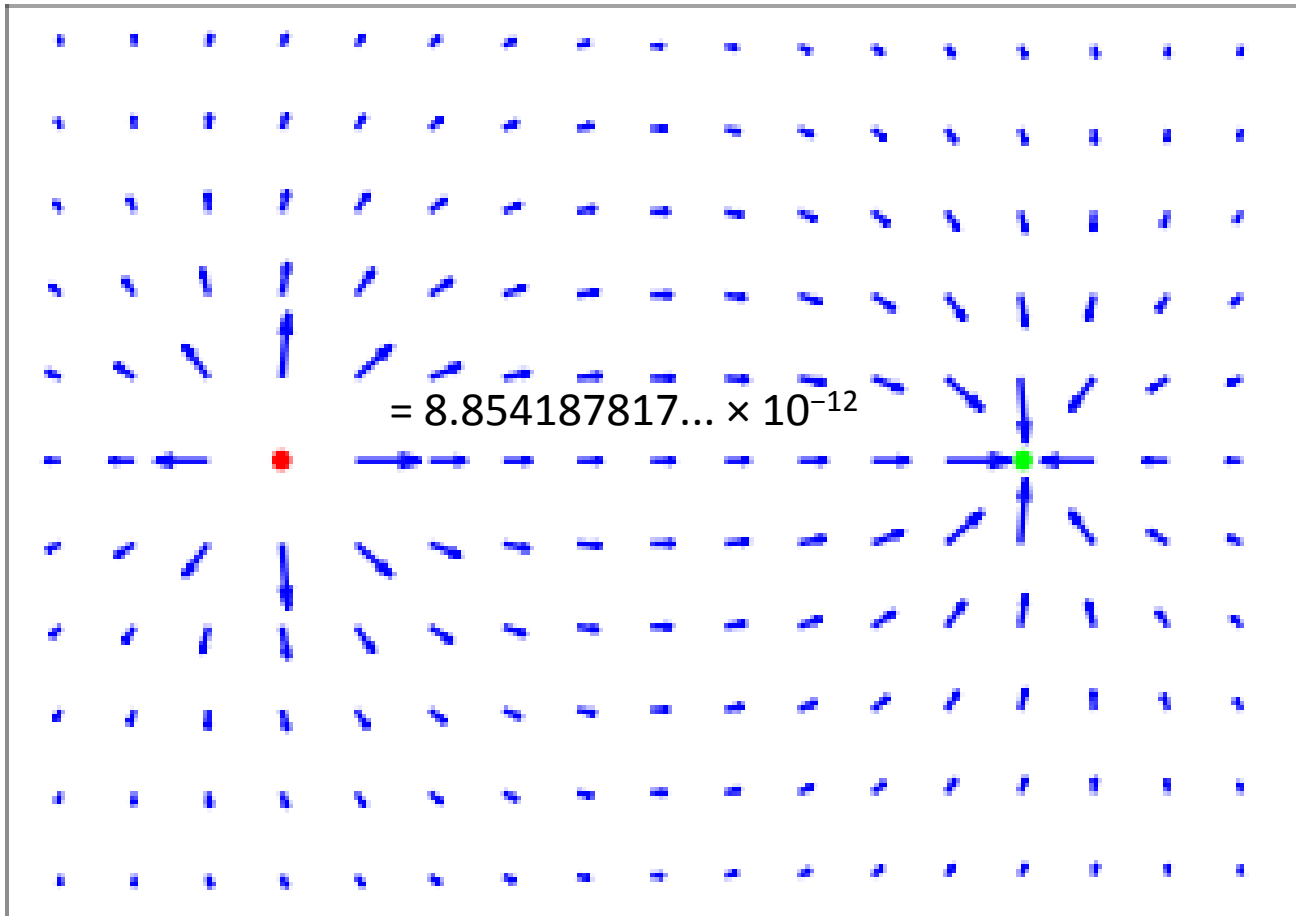
$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{\mathbf{r}} \qquad (1)$$

Similarity to Newtons “Law” of Gravity
Both Coulomb and Newton are inverse square laws

$$\mathbf{F} = G\frac{Mm}{r^2}\hat{\mathbf{r}} = m\mathbf{g}$$

$$\mathbf{F} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}\hat{\mathbf{r}} = q\mathbf{E}$$

Two charges – a dipole



Energy density in the electric field

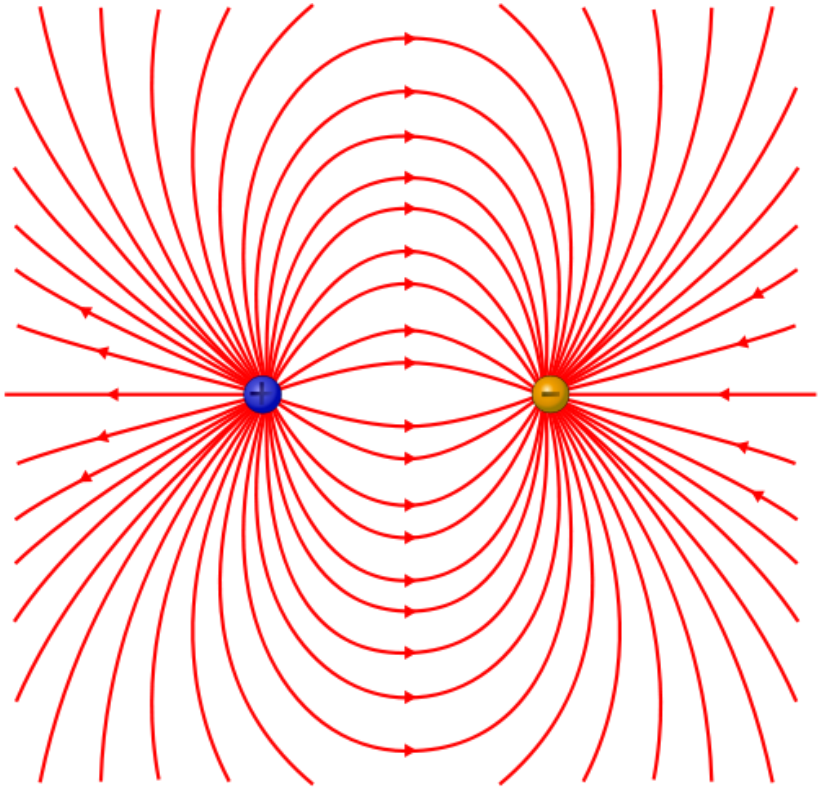
$$u = \frac{1}{2}\varepsilon|\mathbf{E}|^2,$$

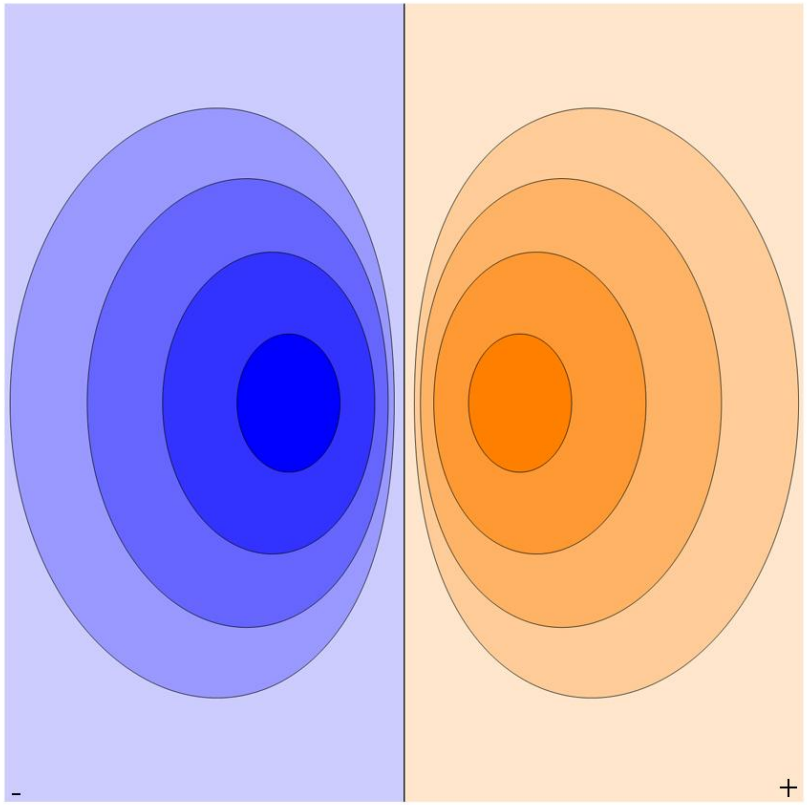
Energy per unit volume J/m^3

$$\frac{1}{2}\varepsilon\int_V |\mathbf{E}|^2\,\mathrm{d}V,$$

Total energy in a volume - Joules

Dipoles
Electric Field Lines - Equipotentials





Dipole moment definition

We define the dipole moment **p** (vector) for a set of charges q_i at vector positions $\mathbf{r}_i$ as:

$$\mathbf{p} = \sum_{i=1}^{N} q_i \mathbf{r}_i \ .$$

For two equal and opposite charges (q) we have p=q*r where r is the distance between them. Vector is from Negative to Positive.

Gauss' Law and Flux

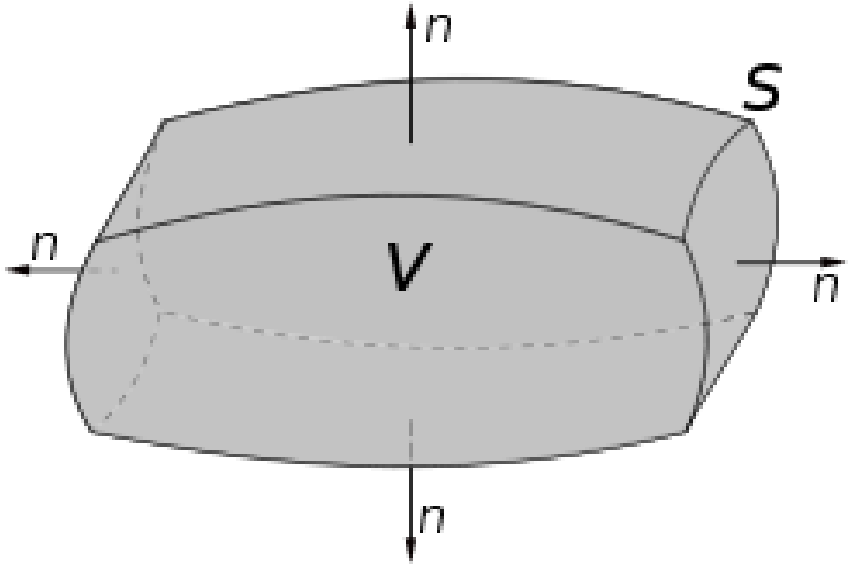
- Lets start by reviewing some vector calculus
- Recall the divergence theorem
- It relates the "flux" of a vector function **F** thru a closed simply connected surface S bounding a region (interior volume) V to the volume integral of the **divergence** of the function **F**
- **Divergence F => ∇●F**

$$\iiint_V (\nabla \cdot \mathbf{F})\, dV = \oiint_S \mathbf{F} \cdot \mathbf{n}\, dS.$$

Volume integral of divergence of **F** = Surface (flux) integral of **F**

Mathematics vs Physics

- There is NO Physics in the previous "divergence theorem" known as Gauss' Law
- It is purely mathematical and applies to ANY well behaved vector field **F**(x,y,z)



Some History – Important to know

- First "discovered" by Joseph Louis Lagrange 1762
- Then independently by Carl Friedrich Gauss 1813
- Then by George Green 1825
- Then by Mikhail Vasilievich Ostrogradsky 1831
- It is known as Gauss' Theorem, Green's Theorem and Ostrogradsky's Theorem
- In Physics it is known as Gauss' "Law" in Electrostatics and in Gravity (both are inverse square "laws")
- It is also related to conservation of mass flow in fluids, hydrodynamics and aerodynamics
- Can be written in integral or differential forms

Gauss' Law in Electromagnetism

- We start with an assumption about the **E** field from a point source.
- Assume it obeys Coulomb's Law – ie inverse square law

$$\mathbf{E}(\mathbf{r}) = \frac{q}{4\pi\epsilon_0}\frac{\mathbf{e_r}}{r^2}$$

Where $\mathbf{e}_r$ is a radial unit vector away from the point charge q

Compute the surface integral of **E(r)** over a sphere of radius r with the charge q at the **center.** We will then use Gauss' Law.

Surface integral over sphere

- Compute the surface integral of **E(r)** over a sphere of radius r with the charge q at the **center.**
- $\int E(r) \bullet dA = 4\pi r^2 * kq/r^2 = 4\pi kq = q/\varepsilon_0$
- (NOTE: **no** r dependence) $k=1/4\pi\varepsilon_0$

- $\nabla \bullet E(r \neq 0) = 0$ – this is true of ANY inverse square field (Gravity also)
- $\nabla \bullet E(r=0) = \delta(r)$ function (∞ at r=0, 0 otherwise)

Coulomb's Law from Gauss' Law

- Assume we have a point charge at the center of a sphere and use Gauss' Law
- And spherical symmetry

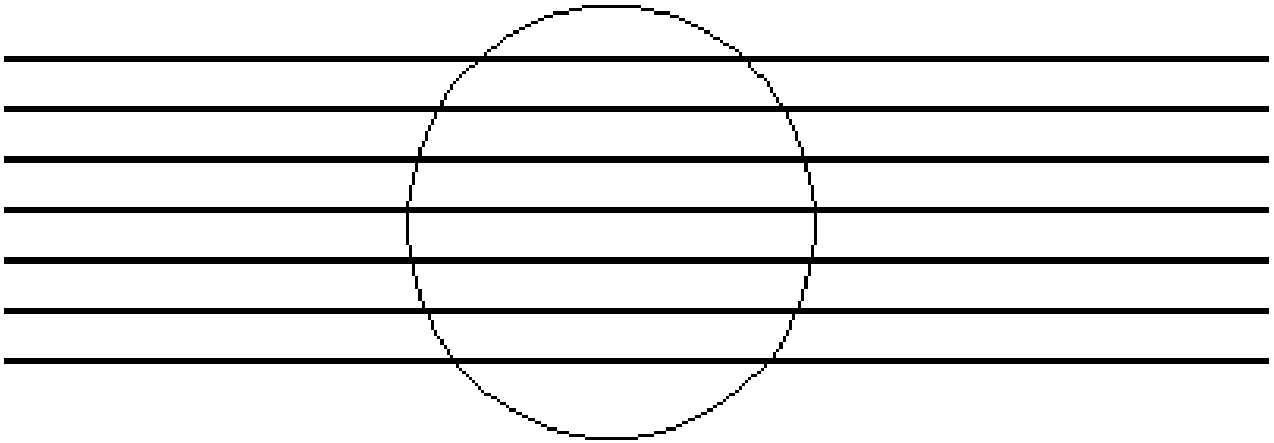
$$\oint_S \mathbf{E} \cdot d\mathbf{A} = Q/\varepsilon_0$$

$$4\pi r^2 \hat{\mathbf{r}} \cdot \mathbf{E}(\mathbf{r}) = Q/\varepsilon_0$$

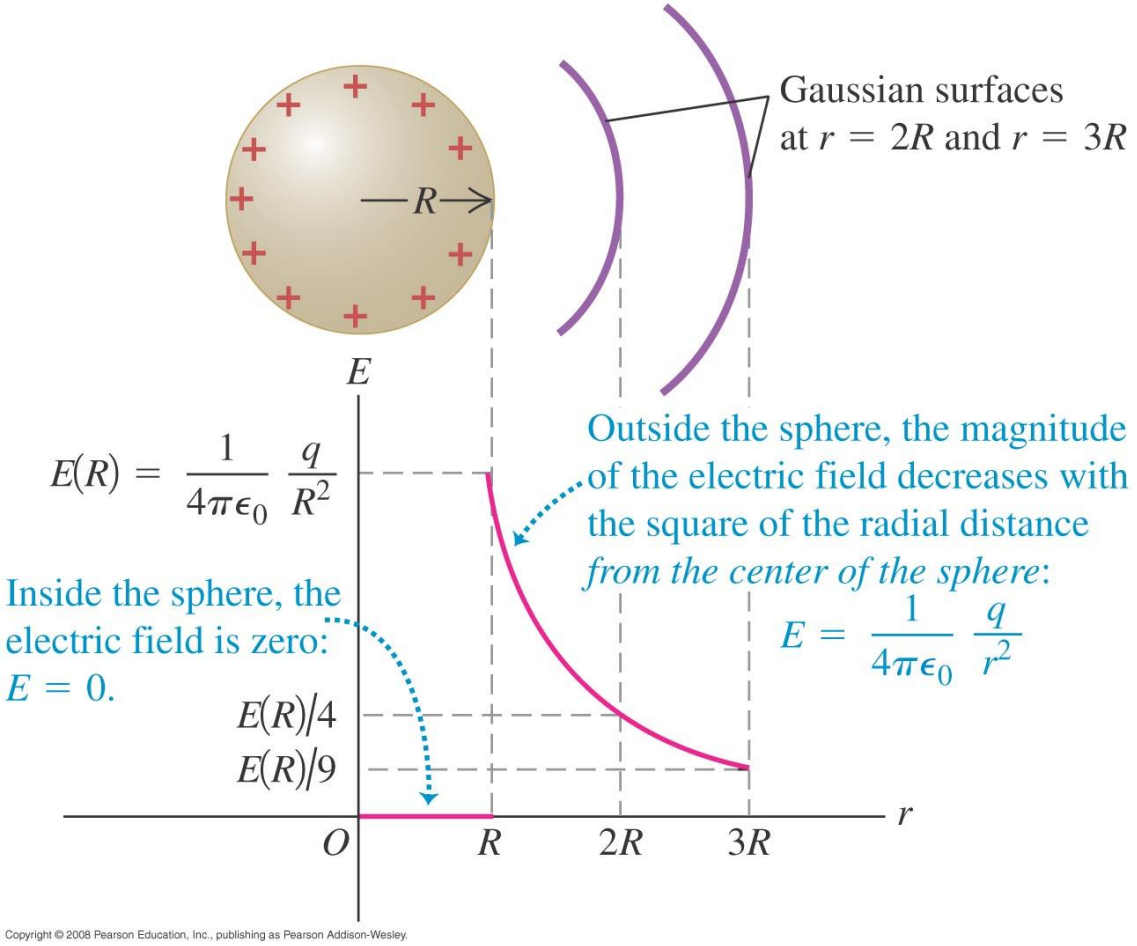
$$\mathbf{E}(\mathbf{r}) = \frac{Q}{4\pi\varepsilon_0} \frac{\hat{\mathbf{r}}}{r^2}$$

Hence we get Coulomb's Law

Flux =0 through sphere

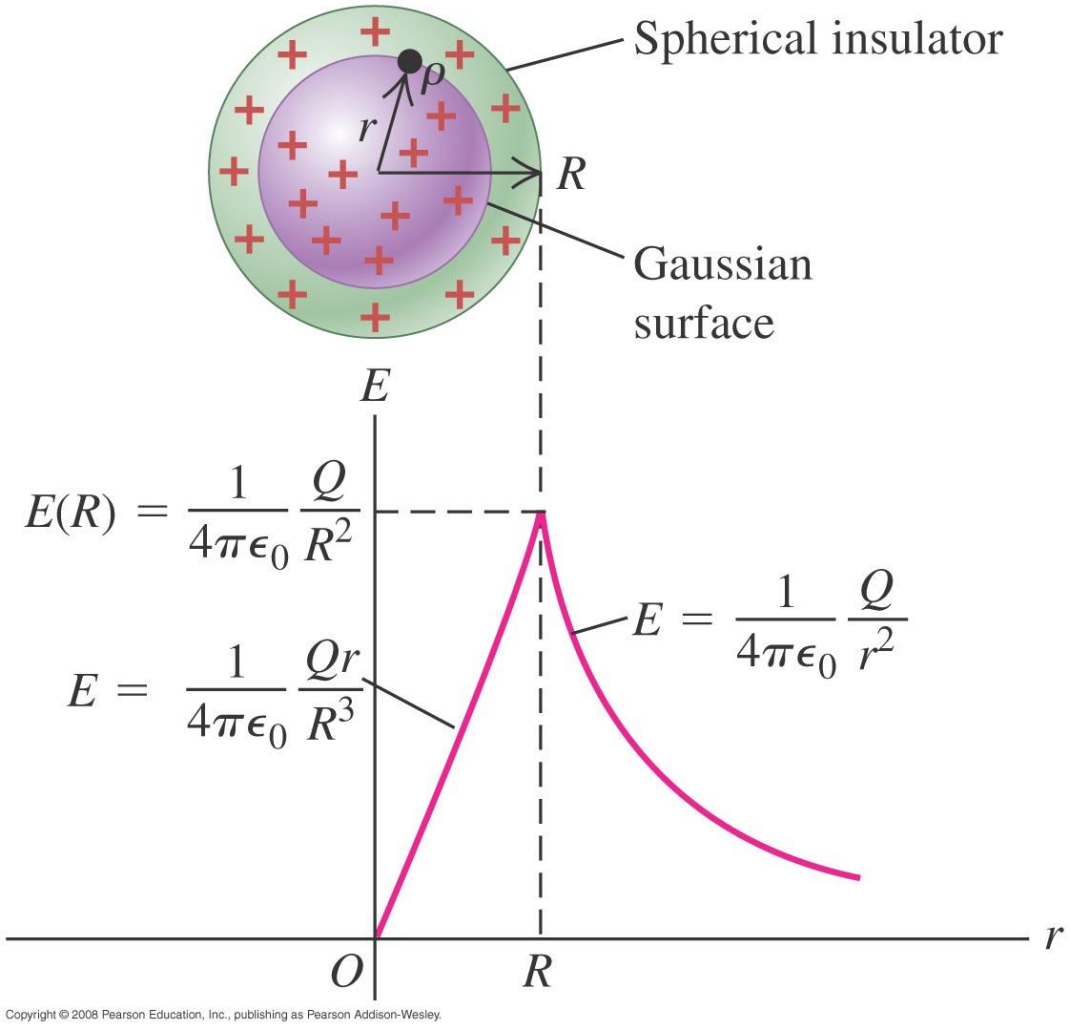


Charged metal sphere – E=0 inside

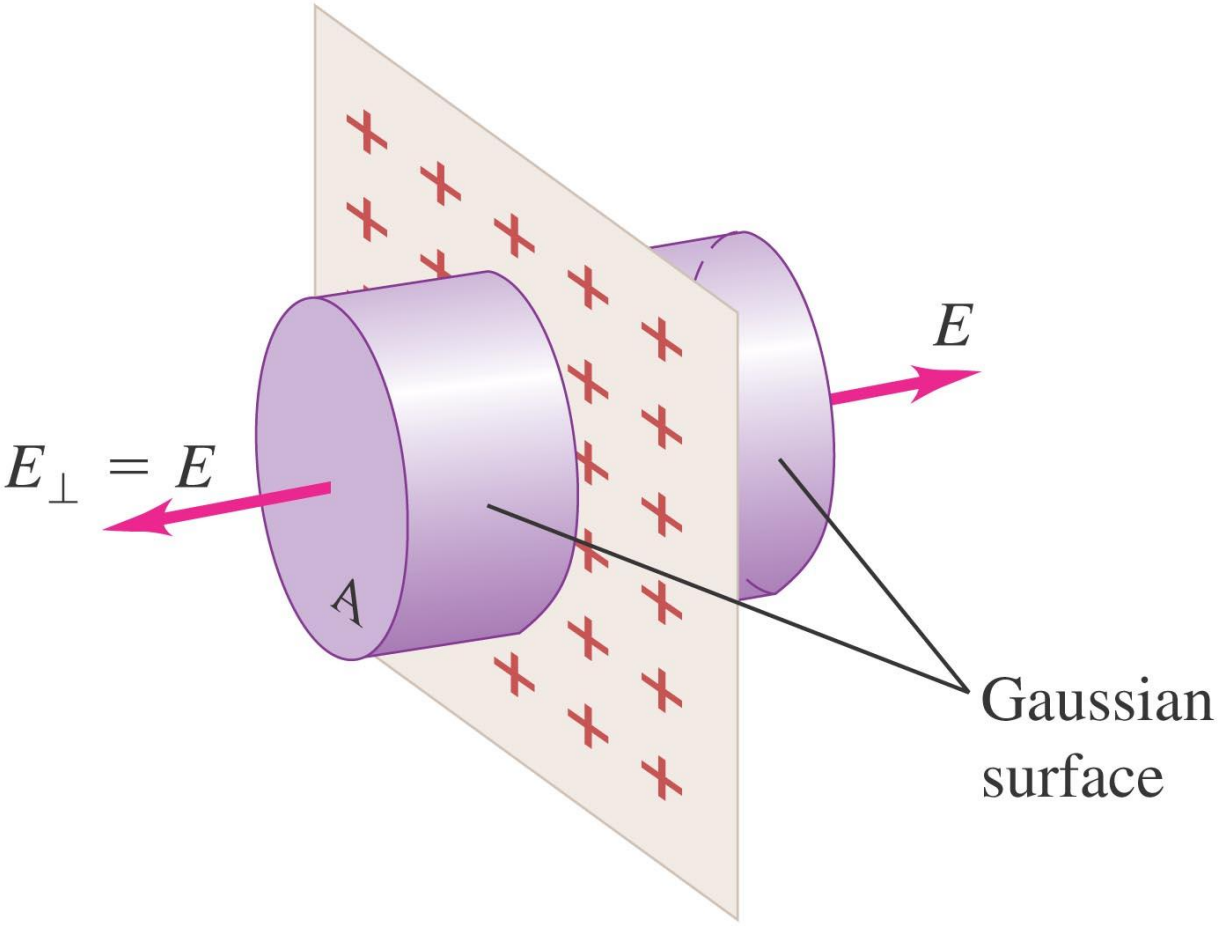


Solving a spherical problem via Gauss' Law

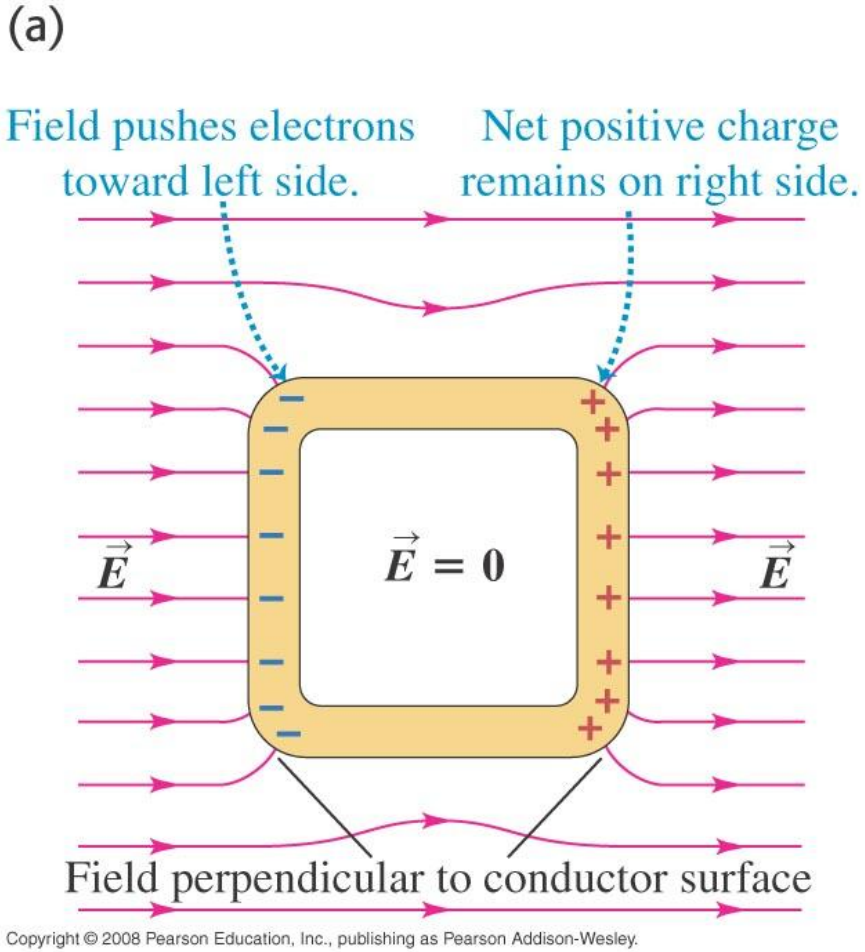
Assume charge Q is spread uniformly over r<R



Gaussian Surface

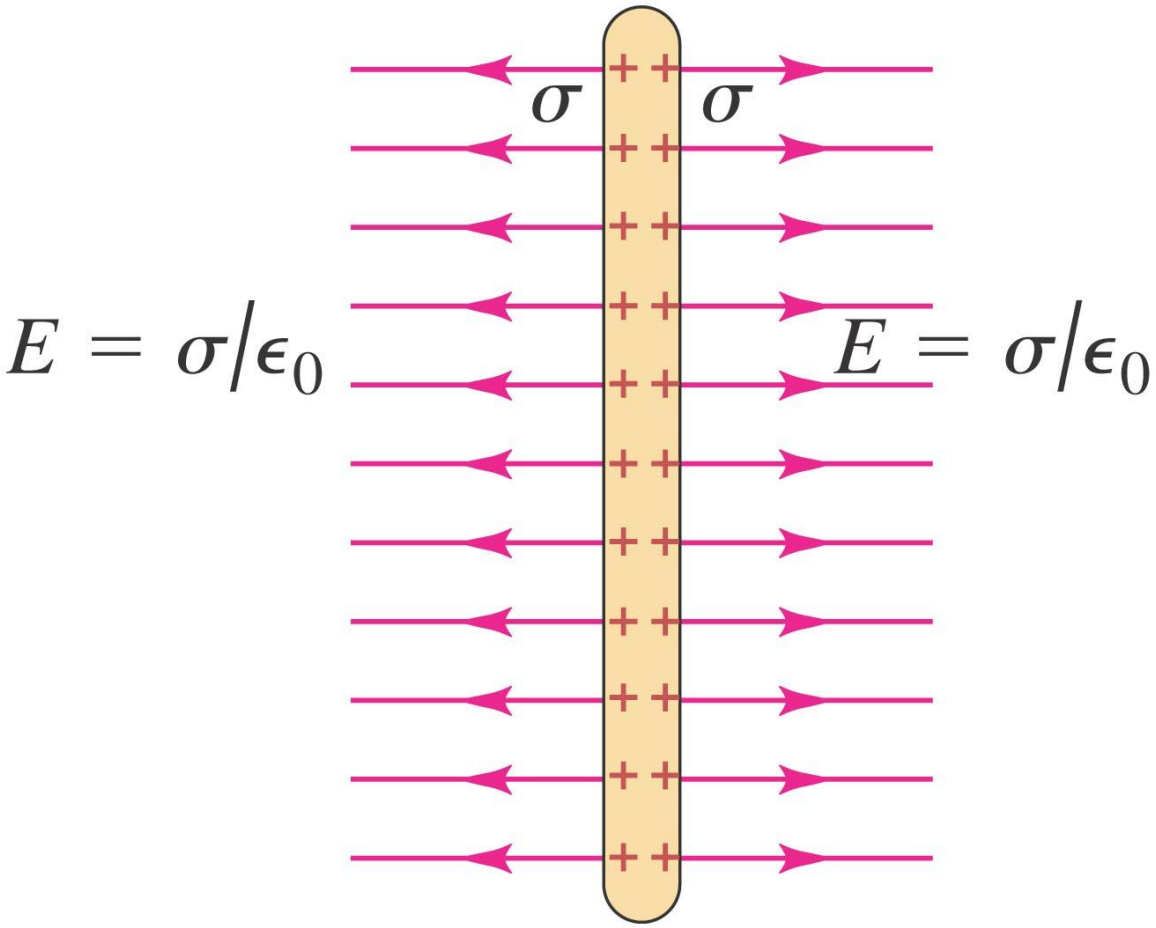


Metal box in external E field
"Faraday Cage"
E=0 inside box

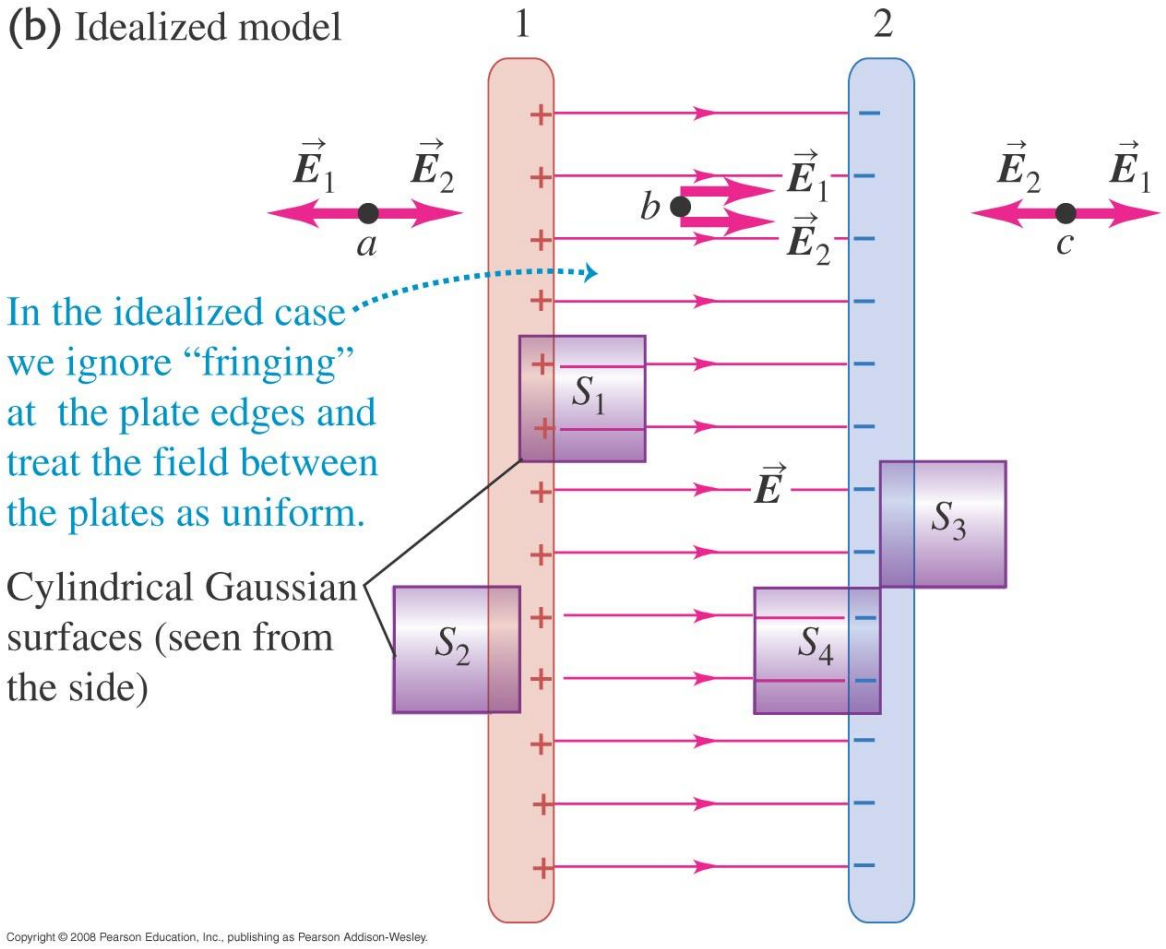


Using a “Gaussian Pillbox” and Gauss’ Law to solve for E field from a uniformly charged metal plate with charge per unit area = σ

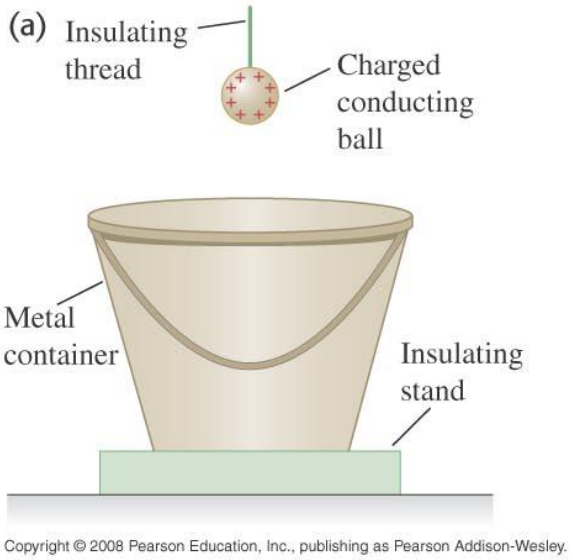
Note E field is the same everywhere except inside metal (=0)

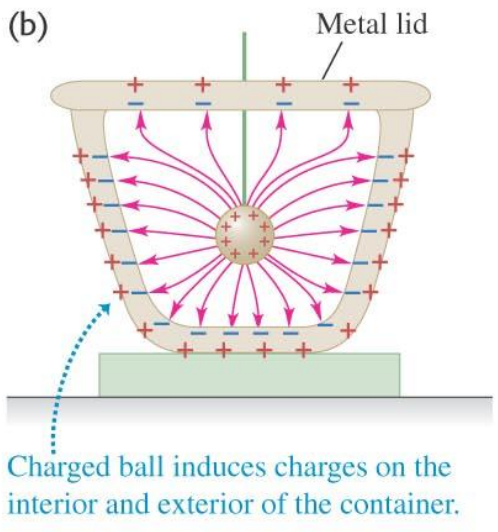


Two metal plates – a Capacitor

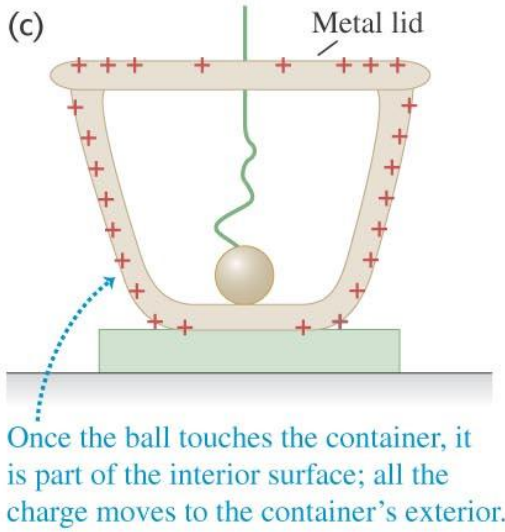


Charged ball and metal container





Charged ball induces charges on the interior and exterior of the container.



Once the ball touches the container, it is part of the interior surface; all the charge moves to the container's exterior.

Lightning

- Approx 16 million lightning storms per year
- Speeds are very high – 60 Km/s (130,000 MPH)!!!
- Temperatures in bolts are very high – can be 30,000 C
- History of kite experiments – wet string = conductive = sparks fly (from key)
- Thomas-François Dalibard and De Lors May 1752
- Benjamin Franklin June 1752 (independent)

Some more on Lightning

- Florida has the most US strikes
- Typ Negative Lightning bolt 30 Kilo amps, 5 Coulombs of charge and 500 Mega Joules of energy
- Large negative bolts can be 120 Kilo amps and 350 Coulombs of charge
- For reference – 1 Ton TNT ~ 4 Giga Joule of energy
- Typ Positive Lightning bolts are 10 times that of Negative Bolts
- Megawatts per meter of bolt are possible
- Typ PEAK power ~ 1 Tera watt (1000 nuclear power plants)
- Lightning heats air to 30,000 C or so and creates supersonic shock wave
- Lightning creates radio waves – these can clear particles from the Van Allen Belts (slots) and create low radiation zones

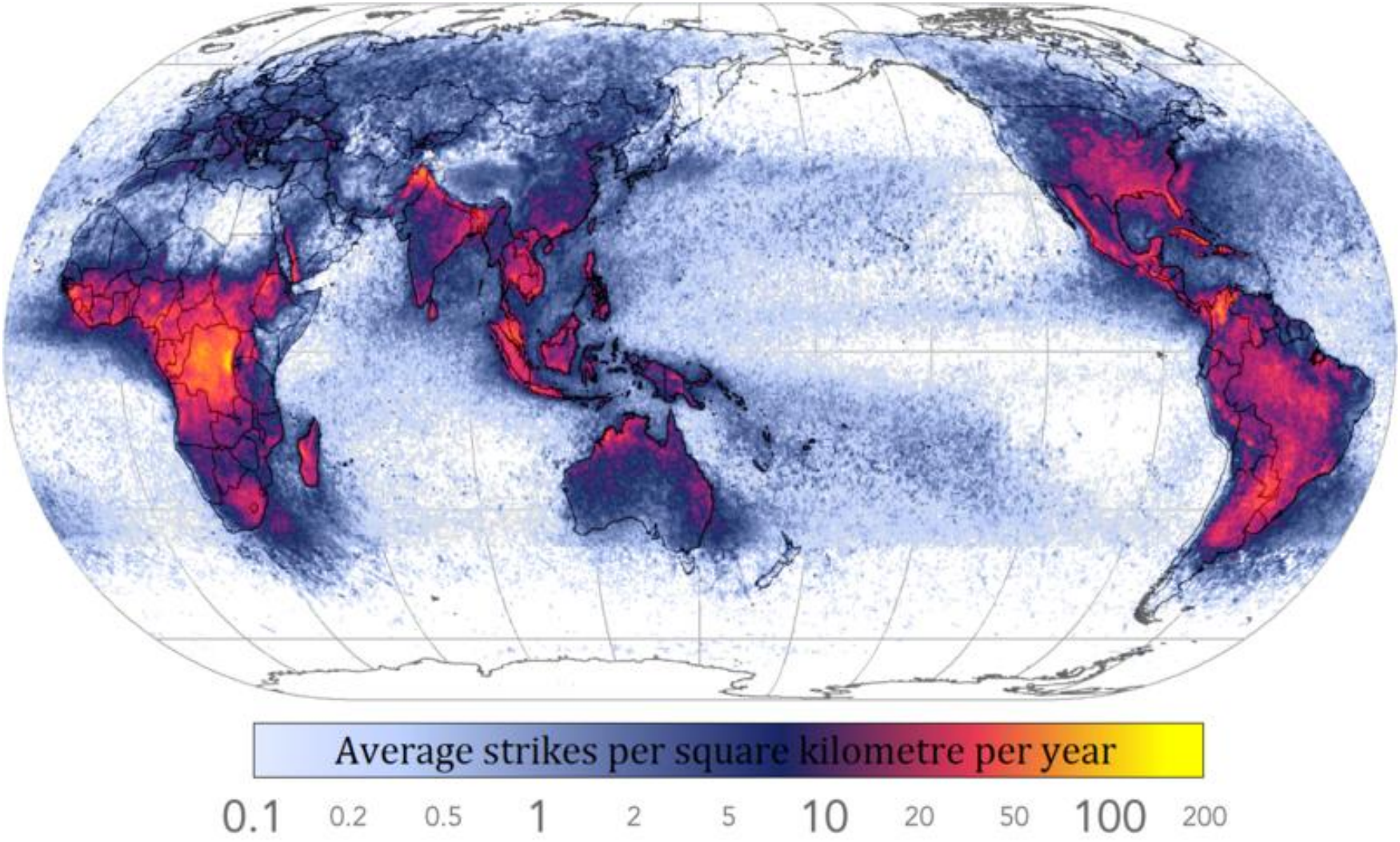
More Lightning Facts

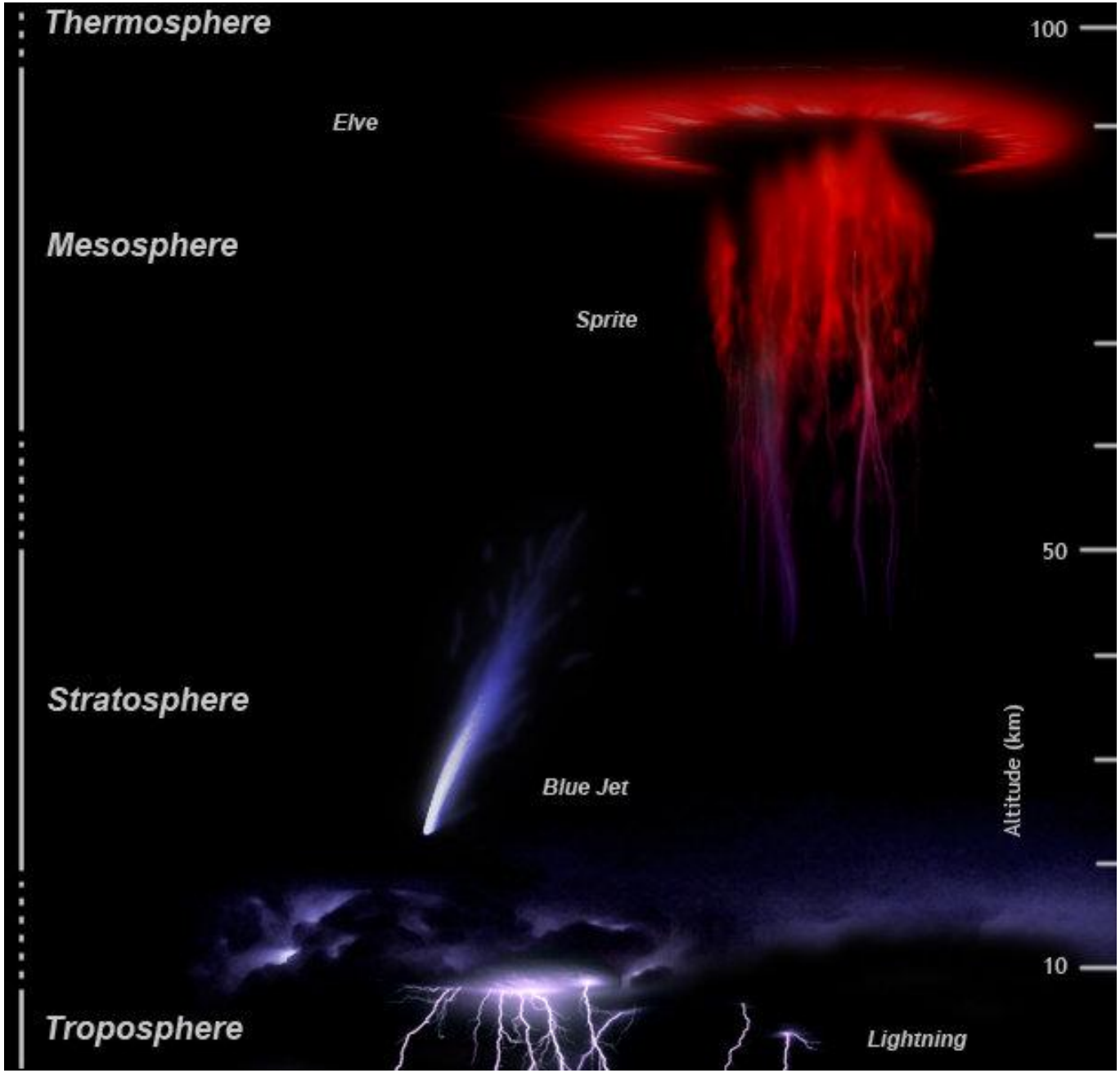
- Norse mythology, Thor is the god of thunder
- Perkūnas - Baltic god of thunder
- Aztec had a god named Tlaloc
- Cyclops gave Zeus the Thunderbolt as a weapon
- Finnish mythology, Ukko (engl. *Old Man*) is the god of thunder
- Brescia, Italy in 1769 – lightning hit the Church of St. Nazaire, igniting 100 tons of gunpoweder that killed 3000 people
- Average lightning hit rate is 44+-5 Hz
- 1.4 billion flashes per year
- Lightning is dangerous - Georg Wilhelm Richmann July 1753 killed as he tried to repeat Ben Franklin's experiment – hit in the head by a blue "ball of lightning"
- Venus, Jupiter and Saturn have lightning

More Lightning Trivia

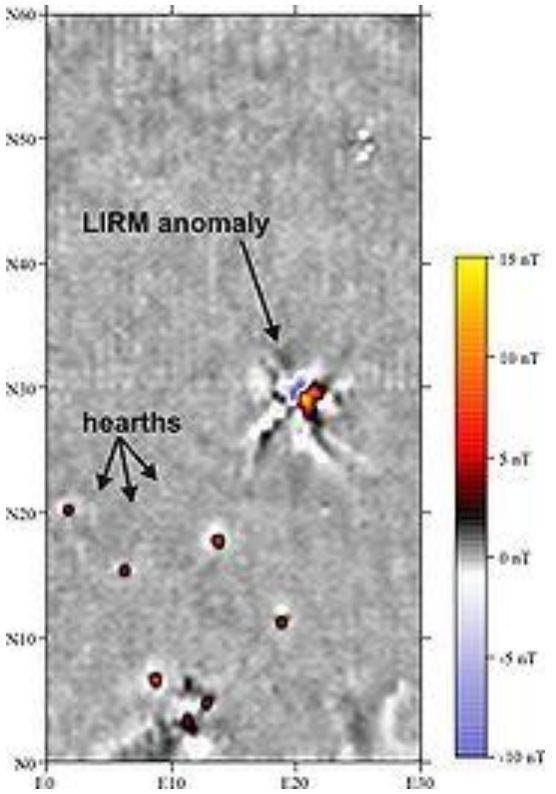
- Terawatt laser in NM induced minor lightning
- Rockets trailing wires can trigger lightning
- **Elves (Emissions of Light and Very Low Frequency Perturbations from Electromagnetic Pulse Sources**) 250 miles up
- Lighning struck Apollo 12 after take off
- Triggered after above ground nuclear testing
- Triggered by volcanoes
- X-Ray, Gamma Ray 20 Mev and anti matter (positrons) seen from lightning
- Lightning strikes can induce ground magnetic “hot spots”
- Roy Sullivan held a Guinness World Record after surviving 7 different lightning strikes across 35 years.
- October 31 2005, sixty-eight dairy cows, died while taking shelter under a tress on a farm at Fernbrook Dorrigo, New South Wales
- December 8, 1963: Pan Am Flight 214 crashed 81 people were killed.
- November 2, 1994, lightning struck fuel tanks in Dronka, Egypt 469 fatalities

Global Lightning strike distribution



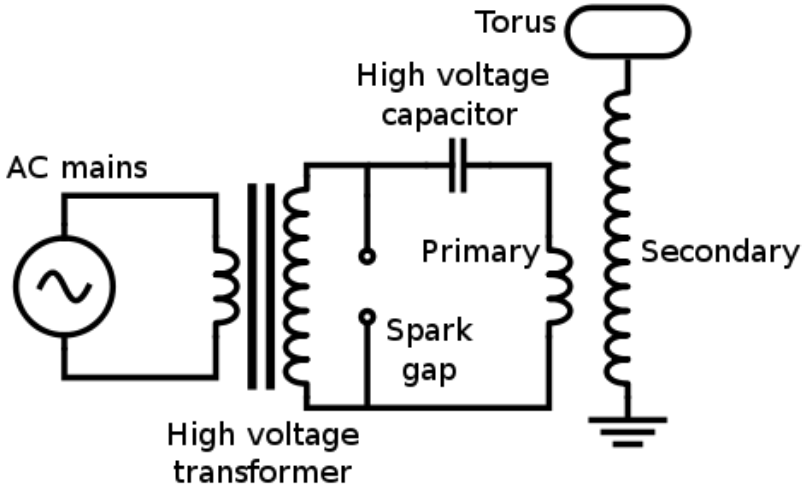

Thermosphere
Elve
Mesosphere
Sprite
Stratosphere
Blue Jet
Troposphere
Lightning
100
50
10
Altitude (km)

Cloud to Cloud



Light strike induced magnetism on the ground

Human Made Lightning – Tesla Coils



Electrostatics – Here it is - dipole moments